## Chapter 9 - Section 1 - Probability

Outcome: the $\qquad$ of an action

Event: a collection of $\qquad$ outcomes

Theoretical Probability: when all outcomes are $\qquad$ likely

$$
\mathrm{P}(\text { event })=\frac{\text { number of favorable events }}{\text { total number of possible outcomes }}
$$

Complement: the collection of outcomes $\qquad$ contained in the event

$$
P(\text { event })+P(\text { not event })=1
$$

## Finding Probability

1) You select a letter at random from the letters shown. Express the following probabilities as a fraction, a decimal, and a percent.
a) $P($ vowel $)=$
b) $P($ constant $)=$

Find Probabilities from 0 to 1
2) The picture shows the jeans in Jason's closet. He selects a pair of jeans with his eyes shut. Find each probability.
a) $P$ (khaki)
b) $P$ (red)

c) $P($ not khaki $)$
3) You roll a number cube once. Find each probability.
a) P (multiple of 3 )

b) $P$ (not a multiple of 2 )
c) $P(9)$

## Challenge-

A bag contains an unknown number of marbles. You know that $\mathrm{P}(\mathrm{red})=\frac{1}{4}$ and $\mathrm{P}($ green $)=\frac{1}{4}$. What can you conclude about how many marbles are in the bag?

## Closure-

1) How do you find the theoretical probability of an event?
2) In a stack are several number cards: there are three 1 s , four 2 s , three 3 s , two 4 s , two 6 s , and six 7 s . You pick a card at random.
a) Write $P(3)$ as a fraction, a decimal, and a percent.
b) Write P (not 7) as a fraction, a decimal, and a percent.

## Bell Ringer-

1) James has 1 blue shirt, 5 white shirts, 3 green shirts, and 2 brown shirts. He selects a shirt from his closet with his eyes shut. Find each probability.
a) $P$ (white shirt)
b) P (not green shirt)
c) $P$ (blue shirt)

## Chapter 9 - Section 2 - Experimental Probability

Experimental Probability: probability based on $\qquad$ data or $\qquad$

$$
\mathrm{P}(\text { event })=\frac{\text { number of times an event occurs }}{\text { total number of trials }}
$$

## Finding Experimental Probability

1) You attempt 16 free throws in a basketball game. Your results are shown. What is the experimental probability of making a free throw?
$P($ free throw $)=$
Results of Free Throw Attempts

| $\mathbf{0}$ |  |  |  |  |  | miss | Make |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |  |  |  |  |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |  |  |  |  |


2) In 60 coin tosses, 25 are tails. Find the experimental probability of getting tails.

P (tails) $=$
3) A bicycle company checks a random sample of bikes. The results are shown below. If the trend continues, which is the best prediction of the number of defective bikes in a batch of 1,300 ?

Quality Control Results

| Defective Bikes | Bikes Checked |
| :---: | :---: |
| 12 | 400 |

a) 430 bikes
b) 390 bikes
c) 43 bikes
d) 39 bikes
4) Predict the number of defective bikes in a batch of 3,500 .
5) Find the experimental probability that 2 of 3 children in a family are girls. Assume that girls and boys are equally likely. Simulate the problem by tossing three coins. Let "heads" represent a girl and "tails" represent a boy. A sample of 20 coins tosses is shown.
$\mathrm{P}($ two girls $)=$

| T T H | T T H | H T H | HT T | H T H |
| :--- | :---: | :---: | :---: | :--- |
| T T H | H H T | H T | TH | H H H |
| H H T | T T H | TH H | HT H | T H T |
| T H T | TH T | THT | HH H | H H H |

## Experimental Probability-

Spin the spinner 20 times. Record you results.
Red-

Blue-

Orange=

Green-


1) $P($ red $)$
2) $P$ (green)
3) $P$ (blue)
4) $P$ (Orange)

Which colors' experimental probability matches its theoretical probability?

|  | Experimental | Theoretical |
| :---: | :---: | :---: |
| $\mathbf{P}$ (red) |  |  |
| $\mathbf{P}$ (blue) |  |  |
| $\mathbf{P}$ (orange) |  |  |
| $\mathbf{P}$ (green) |  |  |

## Challenge-

Find the probability of getting a total of 8 when you roll two standard number cubes.

## Extended Question...

You toss a coin 50 times and get tails 32 times. How does this experimental probability of getting a tail compare to the theoretical probability of getting a tail?
a) the probabilities are the same
b) the experimental probability is lower
c) the experimental probability is higher
d) it cannot be determined

## Closure-

1) How do you find the experimental probability of an event?
2) A manufacturer makes computer circuit boards. A random check of 5,000 circuit boards shows that 25 are defective.
a) What is the experimental probability that a circuit board is defective?
b) Predict the number of defective circuit boards per month if the company manufactures 41,000 circuit boards in July.

## Bell Ringer-

1) A manufacturer of computer parts checks 100 parts each day. On Monday, 2 of the checked parts are defective.
a) What is the experimental probability that a part is defective?
b) Predict the probable number of defective parts in Monday's total production of 1,250 parts.

## Chapter 9 - Section 3 - Sample Spaces

Sample Space: the collection of all $\qquad$ in an experiment

Tree Diagram: a way to show all the $\qquad$ in an experiment

Counting Principle: suppose there are $m$ ways of making one choice and $n$ ways of making a second choice. Then there are $m(n)$ ways to make the first choice followed by the second.


## Finding a Sample Space

1) Fill in the table to find the sample space for rolling two number cubes. Write the outcomes as ordered pairs.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

a) Find the probability of rolling at least one 3 .
b) Find the probability of rolling doubles.
c) Find the probability of rolling a sum of 6 .
2) Fill in the table to find the sample space for tossing two coins. Write the outcomes as ordered pairs.
a) Find the probability of getting two heads.

|  | $H$ | $T$ |
| :---: | :---: | :---: |
| $H$ |  |  |
| $T$ |  |  |

b) Find the probability of getting a head and a tail.

## Using a Tree Diagram

3) Suppose you are going to travel on a river. You have two choices of boats- a kayak or a rowboat. You can go upstream on three smaller streams, to the north, northwest, and northeast.
a) What is the sample space for your journey? Make a tree diagram for the possible outcomes.

b) Suppose you select a trip at random. What is the probability of selecting a kayak and going directly north?
c) Suppose a canoe is added as another choice of boats. Draw a tree diagram to show the new sample space.
d) Find the probability of selecting a canoe at random for the trip.

## Using the Counting Principle

4) How many different sandwiches can you order when you choose one bread and one meat from the menu?

The Deli Sandwiches

| Fresh Breads | Deli Meats |
| :---: | :---: |
| Rye | Roast Beef |
| Wheat | Turkey |
| White | Ham |
| Pita | Pastrami |
| Wrap | Salami |
|  | Liverwurst |

5) A manager at the Deli decides to add chicken to the list of meat choices. How many different sandwiches are now available

## Challenge-

Refer to the number cube, spinner, and cards below. Using the Counting Principle, determine the number of outcomes, then find each probability.


1) You roll the die and spin the spinner. $P(2$, red $)$
2) You roll the die and pick a card. P(even\#, Z)
3) You roll the die, spin the spinner and pick a card. P(number<5, blue, not W)

## Closure-

Draw a tree diagram to show the sample space, then use the tree diagram to answer the following probability questions.


1) The number cube is rolled and the spinner is spun.
a) $P(2$, blue $)$
b) $P(1$ or 3, green $)$
c) $P(7$, purple $)$
d) P(even\#, red or blue)

## Bell Ringer-

Madison can order a small, medium, or large pizza with thick or thin crust. How many possible ways can she order? Draw a tree diagram to find each probability.
a) $P($ Large , Thin crust $)$
b) $P($ Small $)$
c) $P$ (Thick crust)

## Chapter 9 - Section 4 - Compound Events

Compound Event: $\qquad$ events

Independent Events: when one event $\qquad$ the probability of the occurrence of the other event

$$
P(A, \text { then } B)=P(A) \cdot P(B)
$$

Dependent Events: when the occurrence of one event $\qquad$ the probability of the occurrence of the other event

$$
P(A, \text { then } B)=P(A) \cdot P(B \text { after } A)
$$

## Probability of Independent Events

1) You and a friend play a game twice.
a) What is the probability that you win both games? Assume that $\mathrm{P}($ win $)$ is $\frac{1}{2}$ $\mathrm{P}($ win, then win $)=$
b) Find P (win, then lose)

## Probability of Dependent Events

2) You select a card at random from those below. The card has the letter $M$. Without replacing the $M$ card, you select a second card. Find the probability that you select a card with the letter A after you select M.

| $M$ | $T$ | $H$ | M | A | T | I |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| C | S |  |  |  |  |  |

a) $P(A)$
b) You select a T card at random. Without replacing the T card you select a second card.

Find $P(S)$
3) You select a card from a bucket that contains 26 cards lettered A-Z without looking. Without replacing the first card, you select a second one.
a) Find the probability of choosing $C$ and then $M$.
$P(C$, then $M)=$
b) Suppose another 26 cards lettered A-Z are put in the bucket.

Find the probability of choosing $J$ and then $J$.
$P(J$, then $J)=$

There is $\mathbf{1}$ yellow marble, $\mathbf{2}$ green marbles, $\mathbf{4}$ red marbles, and 5 blue marbles in a bag. Once a marble is drawn, it is replaced. Find the probability of each outcome.

1) $P$ (a blue then a red marble)
2) P (two green marbles in a row)

3) $P(a$ yellow then a blue marble)

There is $\mathbf{1}$ yellow marble, $\mathbf{2}$ green marbles, 4 red marbles, and 5 blue marbles in a bag. Once a marble is drawn, it is not replaced. Find the probability of each outcome.
5) $P(a$ red then a blue marble)
6) P (two blue marbles in a row)

7) $P(a$ green then a red marble)
8) $P(a$ blue then 2 yellows in a row)

Meghan has a pack of M\&M's. She has 4 red, 6 blue, 3 yellow, 1 green, 4 orange, 2 brown M\&M's. She puts the M\&M's in a cup and takes one out at a time and eats them. Find each probability.

1) $P($ red, yellow)
2) P (two blues in a row)
3) P(green, orange, brown)
4) P(green, red)

## Challenge

You have two spinners with colors on them. The probability of spinning green on both spinners is $\frac{5}{21}$. The probability of spinning green on the first one alone is $\frac{1}{3}$. What is the probability of spinning green on the second spinner alone.

## Closure-

1) How do you find the probability of a compound event?
2) Explain the difference between independent and dependent events.
3) You roll a number cube twice. What is $P(2$, then even $)$ ?
4) You draw card at random from a stack of ten cards, each labeled with a number from 1 to 10 . Then you draw a second card. What is $\mathrm{P}(5$, then 3$)$ ?

## Bell Ringer-

1) A box contains the same number of green marbles, orange marbles, and blue marbles. You draw one marble, replace it and draw a second marble. What is the probability that both marbles you draw are blue?
2) You select a card at random from those having A, E, I, O, U, P, C. The card has the letter E. Without replacing the E card, you select a second card. Find the probability of selecting a card that does not have a vowel.

## Chapter 9 - Section 5 - Simulating Compound Events

Simulation: a model used to $\qquad$ the probability of an event.

Trial: $\qquad$ of the simulation.

## Designing a Simulation

1) A cereal company marks $\frac{1}{6}$ of its box lids with stars. If a customer gets a star, he or she wins a prize. Design a simulation for estimating the probability that a customer will need to buy at least 3 boxes to win a prize.

Step 1: Choose a simulation tool.
$\frac{1}{6}$ of the boxes are marked with stars, so use a tool that has 6 equally likely outcomes.

Step 2: Decide which outcomes are favorable.
$\frac{1}{6}$ of the outcomes should represent a box with a star.

Step 3: Describe the trial.

Using a Simulation to Estimate Probability
2) Perform 20 trials of the simulation you designed in Example 1. Then estimate the probability that a customer will need to buy at least 3 cereal boxes to win a prize.

| Boxes Needed <br> to Win Prize |
| :--- |
| Frequency |
| 1 |
| 2 |
| 3 or more |


3) One-fourth of the deer in a population has a certain disease. Design a simulation for estimating the probability that a scientist will need to test no more than 3 deer before finding one with the disease.

Step 1: Choose a simulation tool.
$\frac{1}{4}$ of the deer has the disease, so use a tool that has 4-equally likely outcomes

Step 2: Decide which outcomes are favorable.
$\frac{1}{4}$ of the outcomes should represent deer with the disease.

Step 3: Describe the trial.

## Using a Simulation to Estimate Probability

4) Perform 20 trials of the simulation you designed in Example 3. Then estimate the probability that a scientist will need to test no more than 3 deer before finding one that has the disease.

| Deer Tested |
| :---: |
| to Find Disease | Frequency


| 3 or fewer |  |
| :---: | :--- |
| 4 or more |  |



## Using Random Digits as a Simulation Tool

5) In an election, $52 \%$ of votes chose Mayor Garner. Use random digits as a simulation tool to estimate the probability that a reporter will ask more than 2 votes before finding one who voted for Garner.

Use a simulation tool with 100 equally likely outcomes. You can use 2-digit random numbers from 00-99.
52 of the possible outcomes should represent voters for Garner. Use the numbers 00 to 51 .

Each row of random numbers below represents one trial. If either or both numbers are between 00 and 51 , the reporter will need to ask 1 or 2 voters. If neither number is between 00 and 51 , the reporter will need to ask more than 2 voters.

Random Numbers

| 06 | 82 |
| :--- | :--- |
| 80 | 17 |
| 87 | 65 |
| 96 | 96 |
| 60 | 68 |
| 47 | 39 |
| 40 | 31 |
| 66 | 17 |
| 30 | 33 |
| 20 | 68 |

## Challenge-

How would increasing the number of trials affect the reliability of the results in a simulation?

## Closure-

1) In a class election, $46 \%$ of students voted for Elliott Rasmussen. Use the random numbers in the chart as a simulation tool to estimate the probability that a school reporter will have to interview more than 1 student to find one who voted for Elliot.

Random Numbers

| 47 | 06 | 92 |
| :--- | :--- | :--- |
| 22 | 55 | 18 |
| 76 | 39 | 24 |
| 01 | 12 | 29 |
| 45 | 28 | 66 |
| 17 | 33 | 52 |
| 55 | 16 | 22 |
| 19 | 15 | 67 |
| 64 | 31 | 90 |
| 70 | 07 | 82 |

