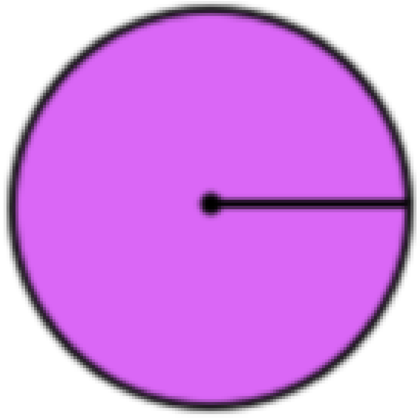
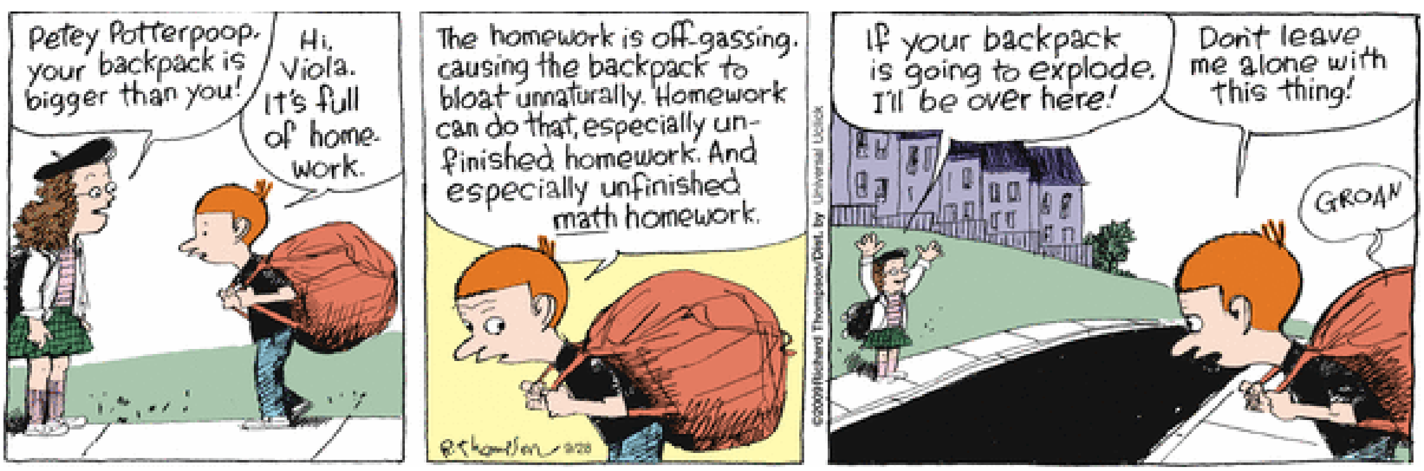
Geometry of the Circle





Name:______________________________

Teacher:____________________________

Pd: _______

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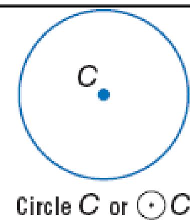
Lines That Intersect Circles – Day 1

Segments in Circles A circle is the locus or set of all points in a plane equidistant from a given point called the center of the circle.

Segments that intersect a circle have special names.



Key Concept **Special Segments in a Circle** For Your FOLDABLE

A **radius** (plural radii) is a segment with endpoints at the center and on the circle.

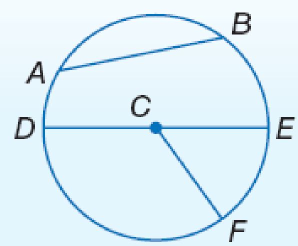
Examples $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of ⊙C.

A **chord** is a segment with endpoints on the circle.

Examples $\overline{AB}$ and $\overline{DE}$ are chords of ⊙C.

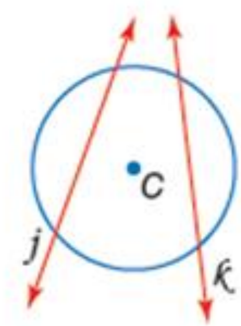
A **diameter** of a circle is a chord that passes through the center and is made up of collinear radii.

Example $\overline{DE}$ is a diameter of ⊙C. Diameter $\overline{DE}$ is made up of collinear radii $\overline{CD}$ and $\overline{CE}$.



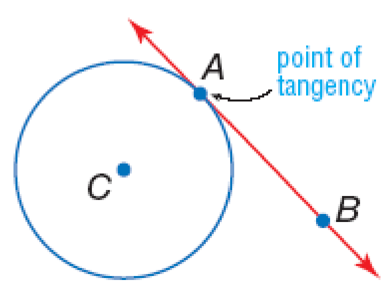
Intersections On or Inside a Circle A secant is a line that intersects a circle in exactly two points. Lines *j* and *k* are secants of ⊙C.

When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.



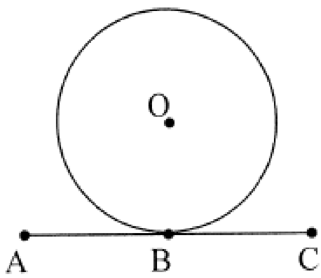
Tangents

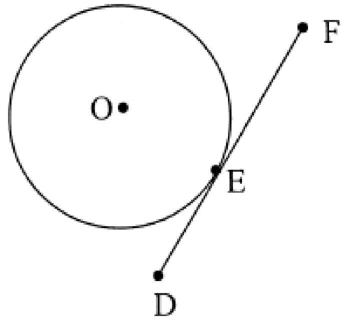
Tangents A tangent is a line in the same plane as a circle that intersects the circle in exactly one point, called the point of tangency. $\overleftrightarrow{AB}$ is tangent to ⊙C at point *A*. $\overline{AB}$ and $\overrightarrow{AB}$ are also called tangents.

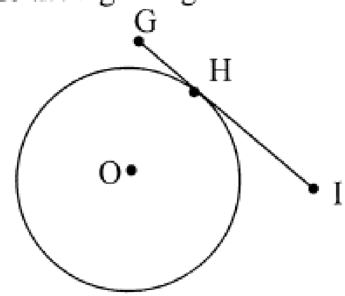


Postulate: if a radius is drawn to the point of contact, then it is perpendicular to the tangent.

1. For the 3 circles below, draw a radius to the point of contact, then name the right angles formed.

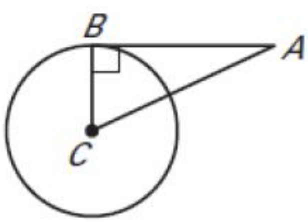






In the diagram, $\overline{BC}$ is a radius of ⊙*C*. Determine whether $\overline{AB}$ is tangent to ⊙*C*. *Explain* your reasoning.

2.



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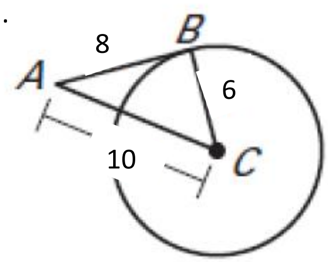

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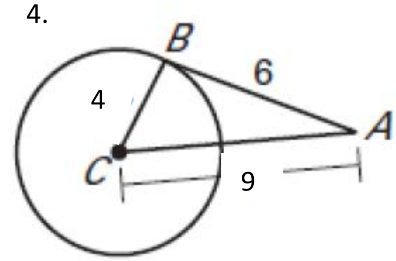
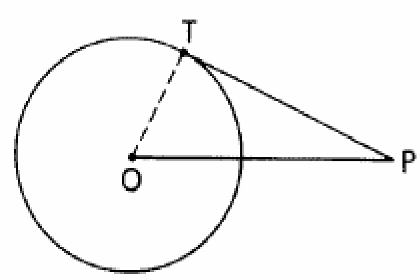
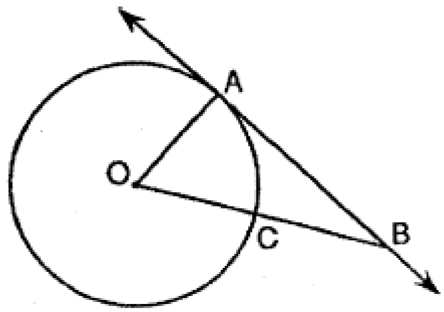


5. $\overleftrightarrow{TP}$ *is tangent to circle O at T.*
The radius of circle O is 8 mm.
Tangent segment $\overline{TP}$ is 6 mm long.
Find the length of $\overline{OP}$.



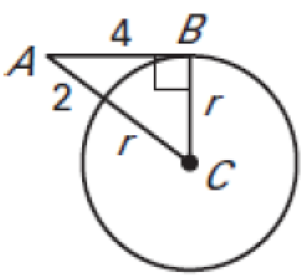
Check Your Progress

In the accompanying diagram, $\overleftrightarrow{BA}$ is tangent to circle O at A. Radii $\overline{OA}$ and $\overline{OC}$ are drawn, and $\overline{OC}$ is extended to intersect $\overleftrightarrow{BA}$ at B. If $BA = 15$ and $OB = 17$, find the measure of a radius of circle O.

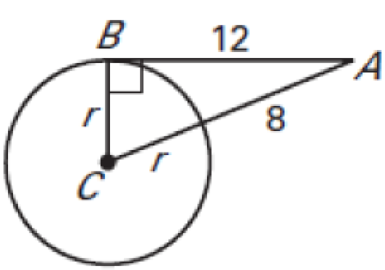


In the diagram, $\overline{AB}$ is tangent to $\odot C$ at point B. Find the radius r of $\odot C$.

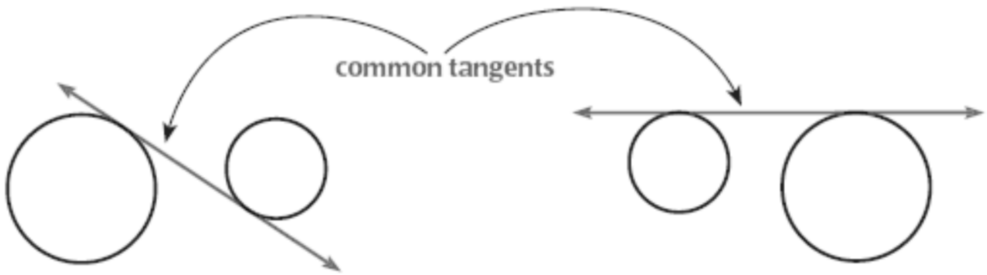
6.



Check Your Progress



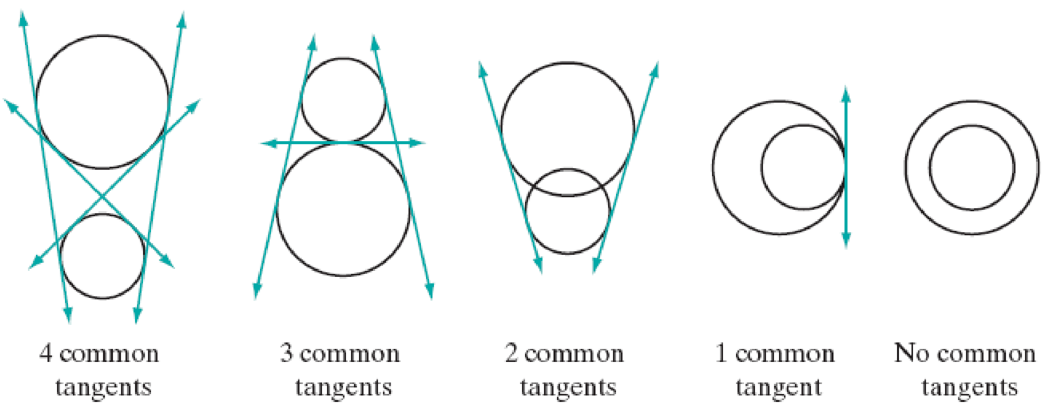
COMMON TANGENTS A line, ray, or segment that is tangent to two coplanar circles is called a *common tangent.*



Common Internal Tangent

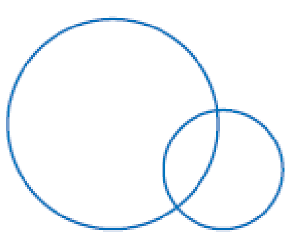
Common External Tangent

The diagrams below show that two circles can have four, three, two, one, or no common tangents.

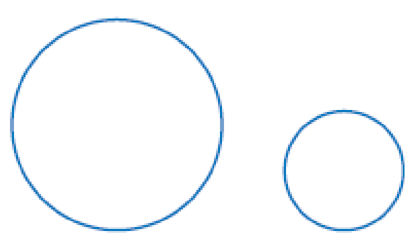


Copy each figure and draw the common tangents. If no common tangent exists, state *no common tangent.*

7.



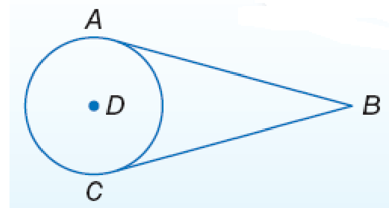
8.



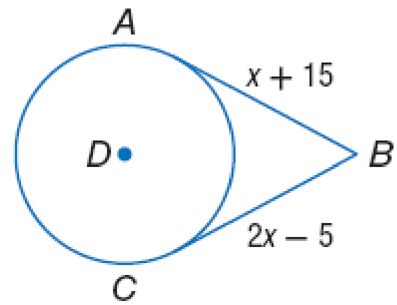
Congruent Tangents

If two segments from the same exterior point are tangent to a circle, then they are congruent.

If $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$, then $\overline{AB} \cong \overline{CB}$.

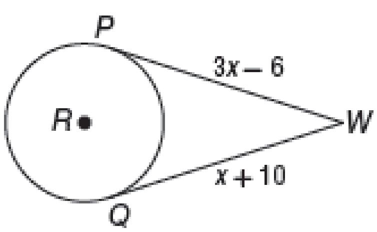


9. **ALGEBRA** $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$. Find the value of x.

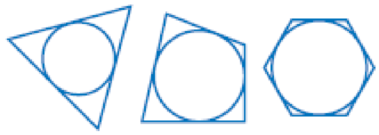
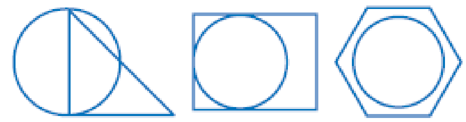


Check Your Progress

Find x. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.

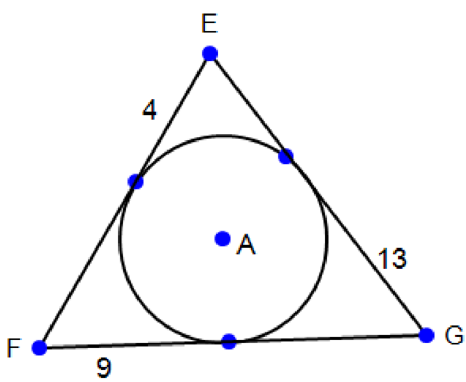


Circumscribed Polygons A polygon is circumscribed about a circle if every side of the polygon is tangent to the circle.

Circumscribed Polygons	Polygons Not Circumscribed
	

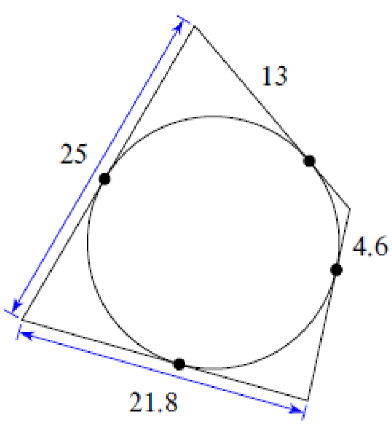
Example 10.

ΔGEF is circumscribed about $\odot A$, find the perimeter of ΔGEF.



Check Your Progress

Find the perimeter of each polygon. Assume that lines which appear to be tangent are tangent.



Challenge

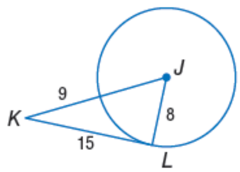
Circle A has diameter 16.4 cm. Circle B has diameter 6.7 cm.

a. If A and B are internally tangent, what is the distance between their centers?

b. If A and B are externally tangent, what is the distance between their centers?

SUMMARY

Identify a Tangent



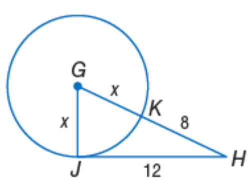
$\overline{JL}$ is a radius of $\odot J$. Determine whether $\overline{KL}$ is tangent to $\odot J$. Justify your answer.

Test to see if $\triangle JKL$ is a right triangle.

$8^2 + 15^2 \stackrel{?}{=} (8 + 9)^2$	Pythagorean Theorem
$289 = 289$ ✓	Simplify.

$\triangle JKL$ is a right triangle with right angle JLK. So $\overline{KL}$ is perpendicular to radius $\overline{JL}$ at point L. Therefore, by Theorem 10.10, $\overline{KL}$ is tangent to $\odot J$.

Use a Tangent to Find Missing Values

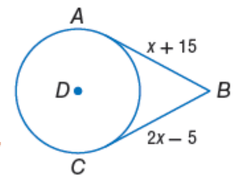


$\overline{JH}$ is tangent to $\odot G$ at J. Find the value of x.

By Theorem 10.10, $\overline{JH} \perp \overline{GJ}$. So, $\triangle GHJ$ is a right triangle.

$GJ^2 + JH^2 = GH^2$	Pythagorean Theorem
$x^2 + 122 = (x + 8)^2$	$GJ = x$, $JH = 12$, and $GH = x + 8$
$x^2 + 144 = x^2 + 16x + 64$	Multiply.
$80 = 16x$	Simplify.
$5 = x$	Divide each side by 16.

Use Congruent Tangents

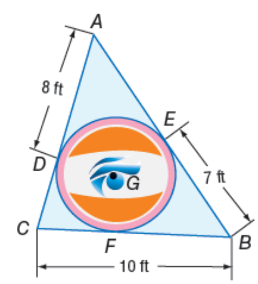


ALGEBRA **$\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$. Find the value of x.**

$AB = CB$	Tangents from the same exterior point are congruent.
$x + 15 = 2x - 5$	Substitution
$15 = x - 5$	Subtract x from each side.
$20 = x$	Add 5 to each side.

Find Measures in Circumscribed Polygons

GRAPHIC DESIGN **A graphic designer is giving directions to create a larger version of the triangular logo shown. If $\triangle ABC$ is circumscribed about $\odot G$, find the perimeter of $\triangle ABC$.**



Step 1 Find the missing measures.

Since $\triangle ABC$ is circumscribed about $\odot G$, $\overline{AE}$ and $\overline{AD}$ are tangent to $\odot G$, as are $\overline{BE}$, $\overline{BF}$, $\overline{CF}$, and $\overline{CD}$. Therefore, $\overline{AE} \cong \overline{AD}$, $\overline{BF} \cong \overline{BE}$, and $\overline{CF} \cong \overline{CD}$.

So, $AE = AD = 8$ feet, $BF = BE = 7$ feet.

By Segment Addition, $CF + FB = CB$, so $CF = CB - FB = 10 - 7$ or 3 feet. So, $CD = CF = 3$ feet.

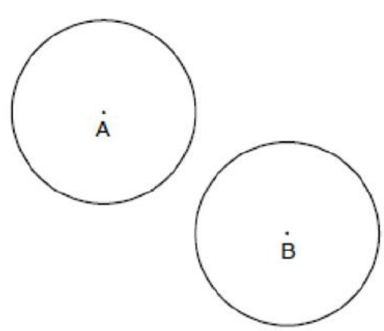
Step 2 Find the perimeter of $\triangle ABC$.

perimeter $= AE + EB + BC + CD + DA$
$= 8 + 7 + 10 + 3 + 8$ or 36

So, the perimeter of $\triangle ABC$ is 36 feet.

Exit Ticket

In the diagram below, circle A and circle B are shown.

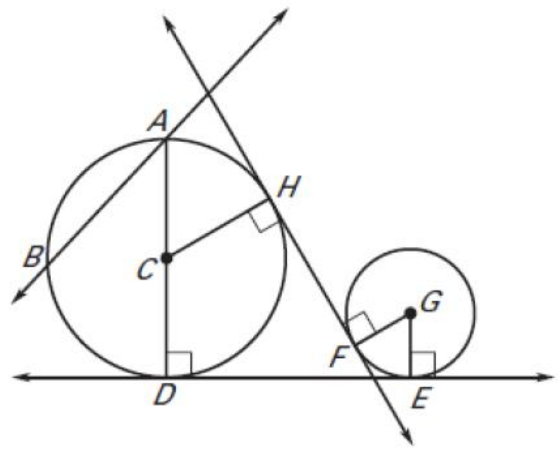


What is the total number of lines of tangency that are common to circle A and circle B?

1) 1
2) 2
3) 3
4) 4

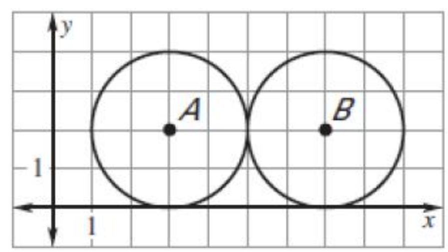
Homework

Match the notation with the term that best describes it.



1.	D	**A.**	Center
2.	$\overleftrightarrow{FH}$	**B.**	Chord
3.	$\overline{CD}$	**C.**	Diameter
4.	$\overline{AB}$	**D.**	Radius
5.	C	**E.**	Point of tangency
6.	$\overline{AD}$	**F.**	Common external tangent
7.	$\overleftrightarrow{AB}$	**G.**	Common internal tangent
8.	$\overleftrightarrow{DE}$	**H.**	Secant

Use the diagram at the right.



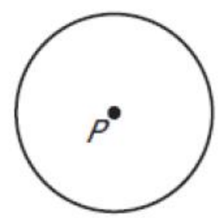
9. What are the diameter and radius of $\odot A$?

10. What are the diameter and radius of $\odot B$?

11. Describe the intersection of the two circles.

12. Describe all the common tangents of the two circles.

Use $\odot P$ to draw the part of the circle described or answer the question.



13. Draw a diameter $\overline{AB}$.

14. Draw tangent line $\overleftrightarrow{CB}$.

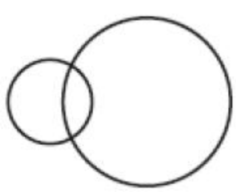
15. Draw chord $\overline{DB}$.

16. Draw a secant through point A.

17. What is the name of a radius in the figure?

Tell how many common tangents the circles have and draw them.

18.



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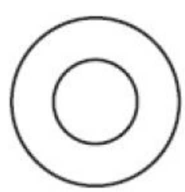

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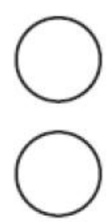
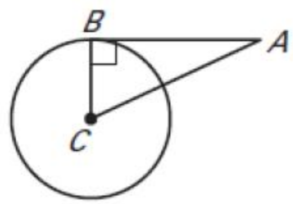


In the diagram, $\overline{BC}$ is a radius of $\odot C$. Determine whether $\overline{AB}$ is tangent to $\odot C$. *Explain* your reasoning.

21.



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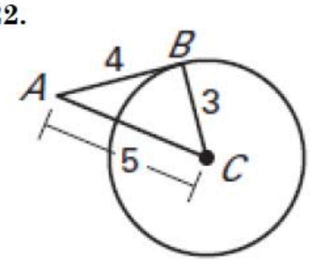
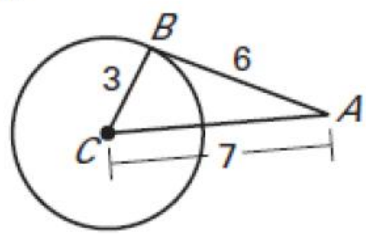
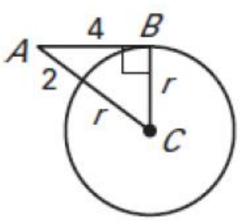


23.

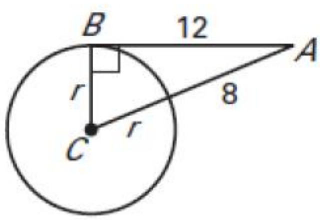


In the diagram, $\overline{AB}$ is tangent to $\odot C$ at point B. Find the radius r of $\odot C$.

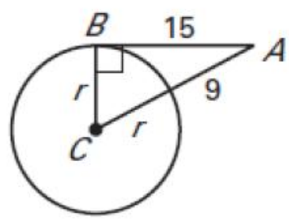
24.



25.

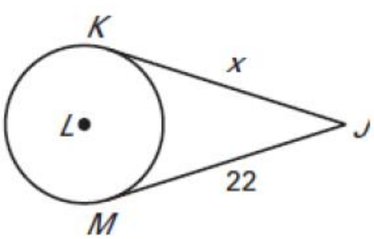


26.

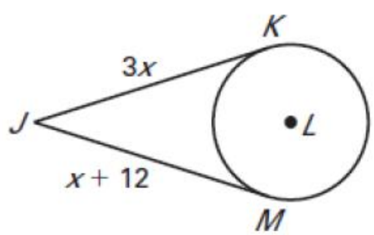


$\overline{JK}$ is tangent to $\odot L$ at K and $\overline{JM}$ is tangent to $\odot L$ at M. Find the value of x.

27.



28.



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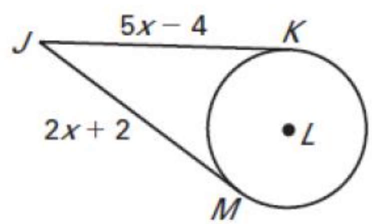
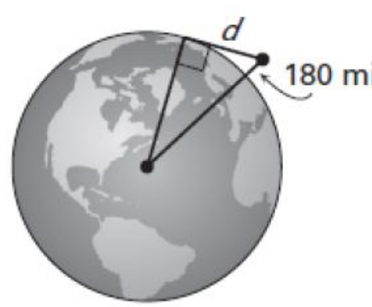


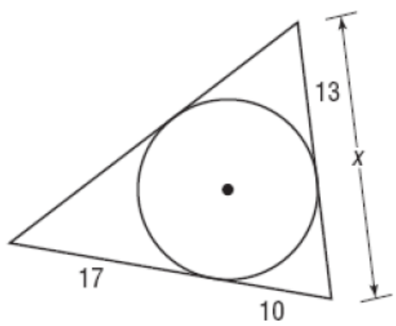
30. Space Shuttle Suppose a space shuttle is orbiting about 180 miles above Earth. What is the distance d from the shuttle to the horizon? The radius of Earth is about 4000 miles. Round your answer to the nearest tenth.



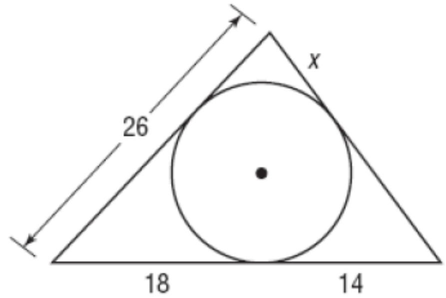
Find the perimeter of each polygon. Assume that lines which appear to be tangent are tangent.

For each figure, find x. Then find the perimeter.

31.



32.



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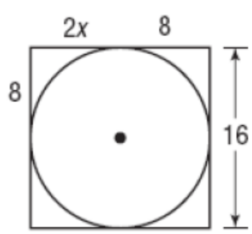

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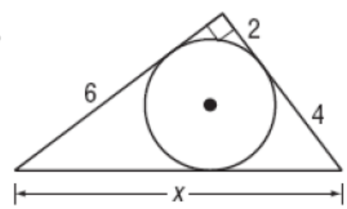

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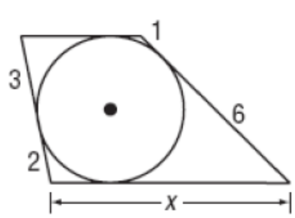

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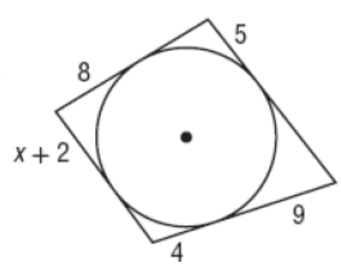
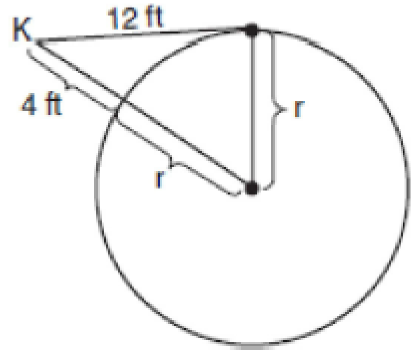


Arcs and Chords of Circles – Day 2

Warm - Up

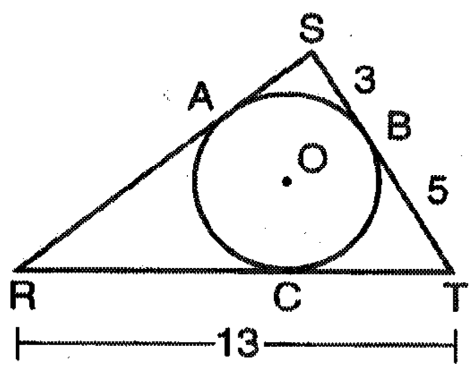
1. Kimi wants to determine the radius of a circular pool without getting wet. She is located at point K, which is 4 feet from the pool and 12 feet from the point of tangency, as shown in the accompanying diagram.



What is the radius of the pool?

1) 16 ft
2) 20 ft
3) 32 ft
4) $4\sqrt{10}$ ft

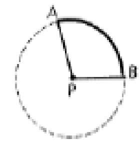
2. In the accompanying diagram, segments $\overline{RS}$, $\overline{ST}$, and $\overline{TR}$ are tangent to circle O at A, B, and C, respectively. If $SB = 3$, $BT = 5$, and $TR = 13$, what is the measure of $\overline{RS}$?



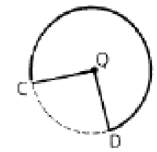
Relating Congruent Arcs, Chords, and Central Angles

Defn: **arc** – 2 points on a circle and all the points in between them.

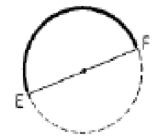
Defn: **minor arc** – an arc whose measure is between 0° and 180°.
[*note*: a minor arc is named with 2 letters – its endpoints]



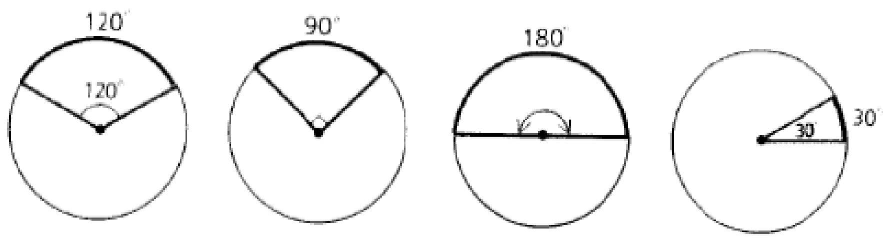
Defn: **major arc** – an arc whose measure is between 180° and 360°.
[*note*: a major arc is named with 3 letters – its 2 endpoints and any point in between]



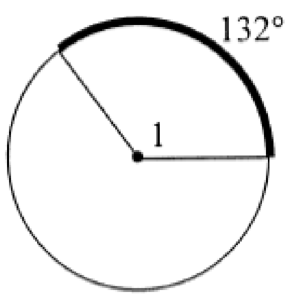
Defn: **semi-circle** – an arc whose measure is exactly 180°.
[*note*: semi-circles should be named with 3 letters like a major arc]



Defn: **central angle** – an angle whose vertex lies in the center of a circle. Its sides are radii.
[*note*: the measure of a central angle is equal to the measure of the arc it intercepts]



1. What is the measure of $\angle 1$?



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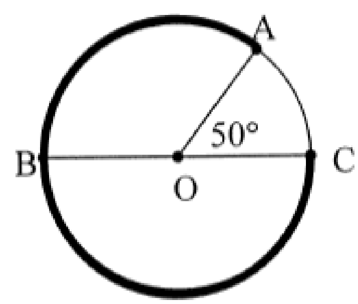
2. What is $m\widehat{ABC}$?



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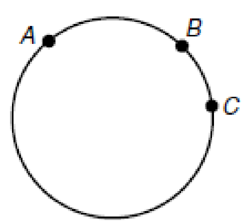
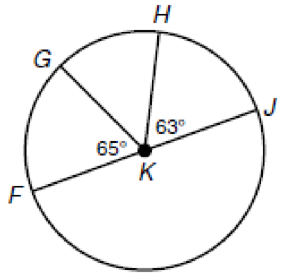
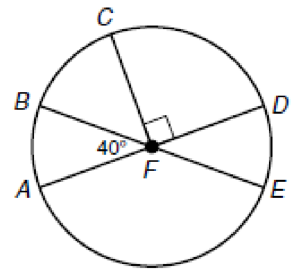
Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. $m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$



Find each measure.





1. m$\widehat{HJ}$ ______________________________

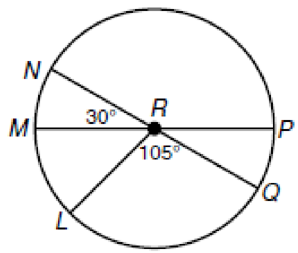
2. m$\widehat{FGH}$ ______________________________

3. m$\widehat{CDE}$ ______________________________

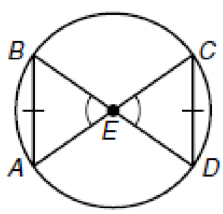
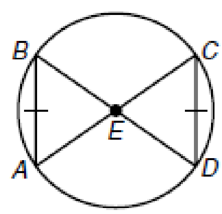
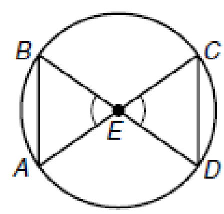
4. m$\widehat{BCD}$ ______________________________

5. m$\widehat{LMN}$ ______________________________

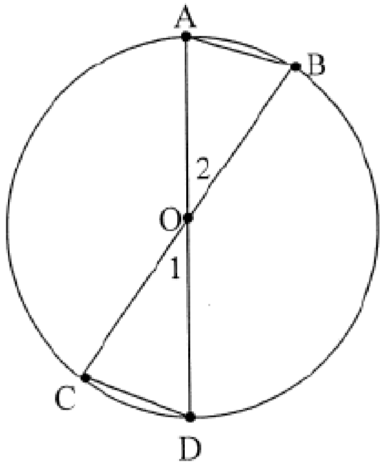
6. m$\widehat{LNP}$ ______________________________



Congruent arcs are arcs that have the same measure.

Congruent Arcs, Chords, and Central Angles		
 If $m\angle BEA \cong m\angle CED$, then $\overline{BA} \cong \overline{CD}$.	 If $\overline{BA} \cong \overline{CD}$, then $\widehat{BA} \cong \widehat{CD}$.	 If $\widehat{BA} \cong \widehat{CD}$, then $m\angle BEA \cong m\angle CED$.
Congruent central angles have congruent chords.	Congruent chords have congruent arcs.	Congruent arcs have congruent central angles.

6 Theorems Summarized



3. If $\angle 1 \cong \angle 2$, then $\overline{AB} \cong$ ______ and $\overset{\frown}{AB} \cong$ _______.

4. If $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong$ ______ and $\angle 1 \cong$ _______.

5. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong$ ______ and $\angle 1 \cong$ _______.

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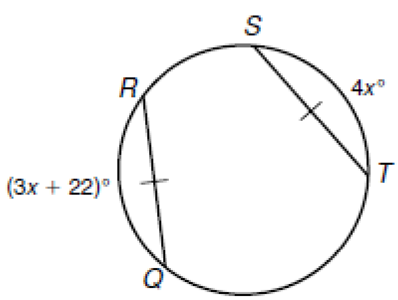
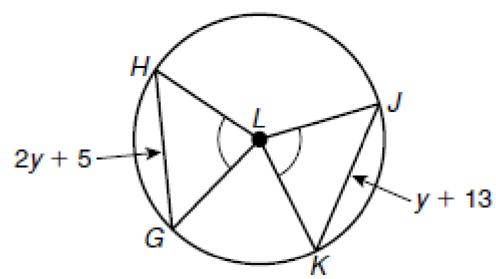
7. $\overline{QR} \cong \overline{ST}$. Find $m\overset{\frown}{QR}$.

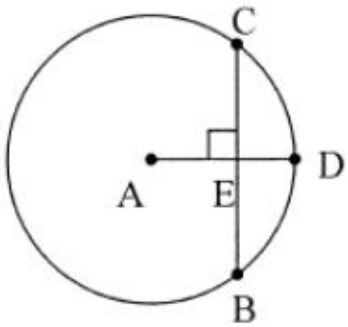


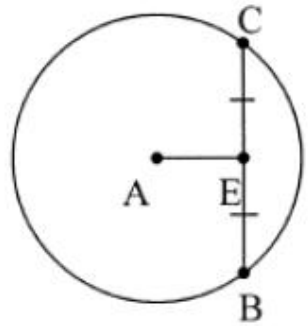
8. $\angle HLG \cong \angle KLJ$. Find GH.



Radius-Chord Relationships

Theorem: If a radius (or part of a radius) is ⊥ to a chord, then it bisects the chord.
Theorem: If a radius (or part of a radius) bisects a chord, then it is ⊥ to the chord.





Complete: If $\overline{AD} \perp \overline{BC}$, then

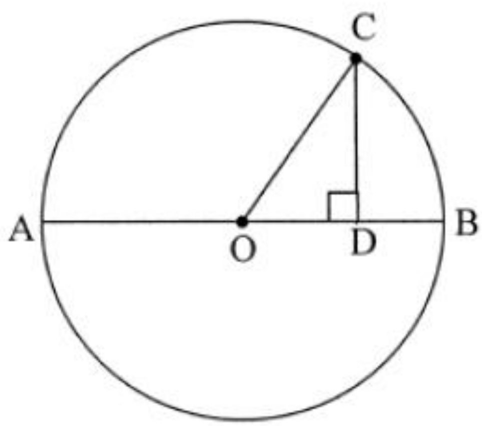
_______ ≅ _______.

Complete: If $\overline{CE} \cong \overline{BE}$, then

_______ ⊥ _______.

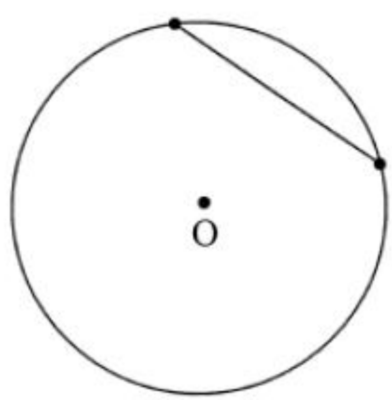
1. Given: ⊙O
   AO = 5
   DB = 2

   Find: OC
   OD
   CD

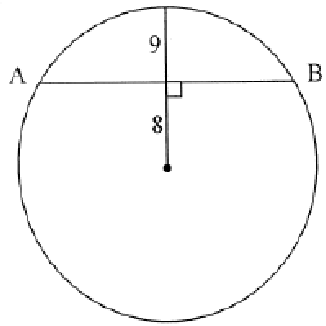


2. Given: ⊙O; chord shown = 12
   radius = 10

   Find: How far is the chord from center?

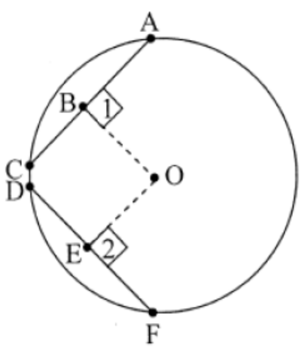


3. How long is AB?



Chords Equidistant from Center

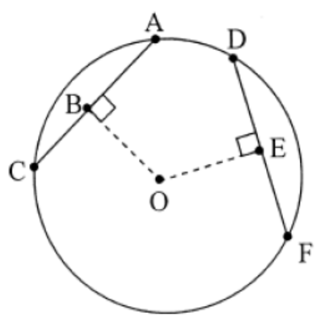
Theorem: If 2 chords are equidistant from the center of a circle, then they are congruent.



Complete: If OB = OE, then

_______ ≅ _______

Theorem: If 2 chords are congruent, then they are equidistant from the center.

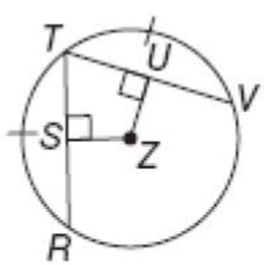


Complete: If $\overline{AC} \cong \overline{DF}$, then

_______ ≅ _______.

Check Your Progress

In $\odot Z$, $\overparen{TR} \cong \overparen{TV}$, $SZ = x + 4$, and $UZ = 2x - 1$. What is x?

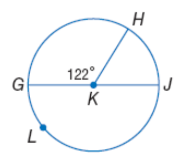


Challenge

Two circles intersect and have a common chord that is 16cm long. The centers are 21cm apart. If the radius of one circle is 10cm, find the radius of the other circle.

SUMMARY

$\overline{GJ}$ is a diameter of $\odot K$. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.



a. $m\widehat{GH}$

$\widehat{GH}$ is a minor arc, so $m\widehat{GH} = m\angle GKH$ or 122.

b. $m\widehat{GLH}$

$\widehat{GLH}$ is a major arc that shares the same endpoints as minor arc $\widehat{GH}$.

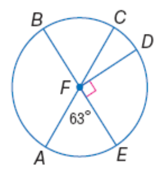
$m\widehat{GHL} = 360 - m\widehat{GH}$

$= 360 - 122$ or 238

c. $m\widehat{GLJ}$

$\widehat{GLJ}$ is a semicircle, so $m\widehat{GLJ} = 180$.

Find each measure in $\odot F$.



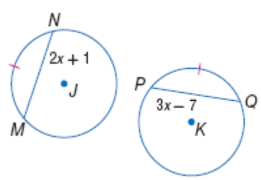
a. $m\widehat{AED}$

$m\widehat{AED} = m\widehat{AE} + m\widehat{ED}$	**Arc Addition Postulate**
$= m\angle AFE + m\angle EFD$	**$m\widehat{AE} = m\angle AFE$, $m\widehat{ED} = m\angle EFD$**
$= 63 + 90$ or 153	**Substitution**

b. $m\widehat{ADB}$

$m\widehat{ADB} = m\widehat{AE} + m\widehat{EDB}$	**Arc Addition Postulate**
$= 63 + 180$ or 243	**$\widehat{EDB}$ is a semicircle, so $m\widehat{EDB} = 180$.**

ALGEBRA In the figures, $\odot J \cong \odot K$ and $\widehat{MN} \cong \widehat{PQ}$. Find PQ.



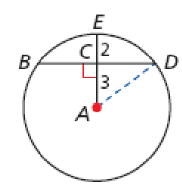
$\widehat{MN}$ and $\widehat{PQ}$ are congruent arcs in congruent circles, so the corresponding chords $\overline{MN}$ and $\overline{PQ}$ are congruent.

$MN = PQ$	**Definition of congruent segments**
$2x + 1 = 3x - 7$	**Substitution**
$8 = x$	**Simplify.**

So, $PQ = 3(8) - 7$ or 17.

Using Radii and Chords

Find BD.



Step 1 Draw radius $\overline{AD}$.

$AD = 5$ — *Radii of a ⊙ are ≅.*

Step 2 Use the Pythagorean Theorem.

$CD^2 + AC^2 = AD^2$

$CD^2 + 3^2 = 5^2$ — *Substitute 3 for AC and 5 for AD.*

$CD^2 = 16$ — *Subtract 3^2 from both sides.*

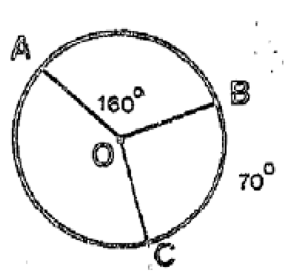
$CD = 4$ — *Take the square root of both sides.*

Step 3 Find BD.

$BD = 2(4) = 8$ — *$\overline{AE} \perp \overline{BD}$, so $\overline{AE}$ bisects $\overline{BD}$.*

Exit Ticket

In the diagram of circle O, radii OA, OB, and OC are drawn. If $m\angle AOB = 160°$ and $m\widehat{BC} = 70°$, find $m\angle AOC$.

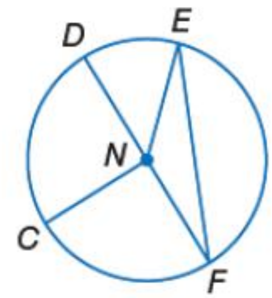


A. 20° C. 130°

B. 110° D. 450°

Day 2 – Homework

For Exercises 1–4, refer to $\odot N$.



1. Name the circle.

2. Identify each.

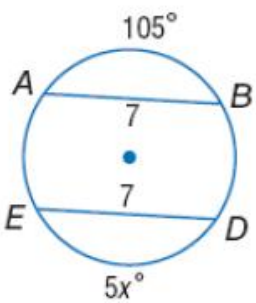
a. a chord **b.** a diameter **c.** a radius

3. If $CN = 8$ centimeters, find DN.

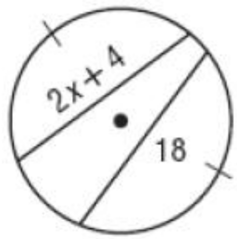
4. If $EN = 13$ feet, what is the diameter of the circle?

ALGEBRA **Find the value of x.**

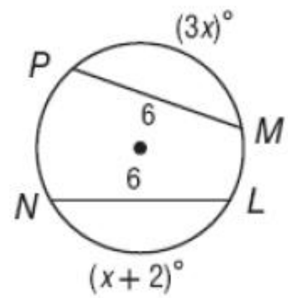
7.



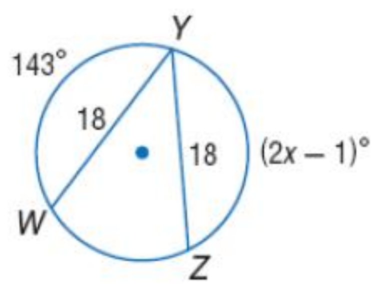
8.



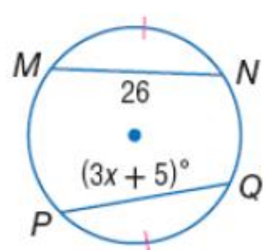
9.



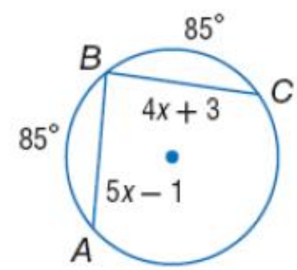
10.



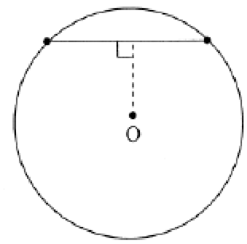
11.



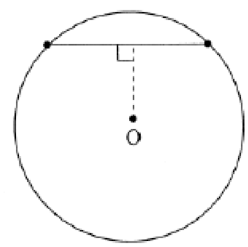
12.



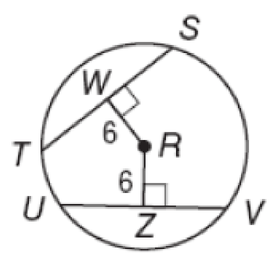
13. If the radius of circle O is 13 and chord MK is 5 cm from the center of the circle, find the length of the chord.



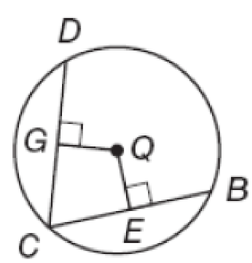
14. Suppose a chord of a circle is 16 inches long and is 6 inches from the center of the circle. Find the length of a radius.



15. In $\odot R$, $TS = 21$ and $UV = 3x$. What is x?

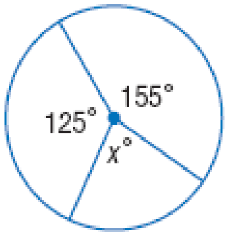


16. In $\odot Q$, $\overline{CD} \cong \overline{CB}$, $GQ = x + 5$ and $EQ = 3x - 6$. What is x?

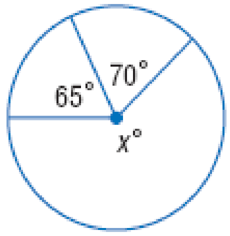


Find the value of x.

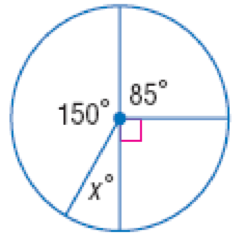
17.



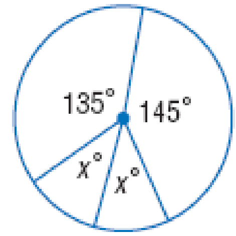
18.



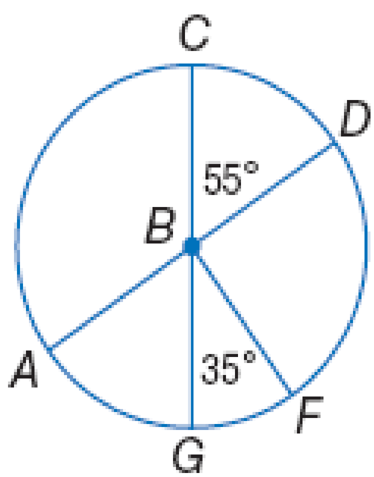
19.



20.



$\overline{AD}$ and $\overline{CG}$ are diameters of $\odot B$. Identify each arc as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.



21. $m\widehat{CD}$

22. $m\widehat{AC}$

23. $m\widehat{CG}$

24. $m\widehat{CGD}$

25. $m\widehat{GCF}$

26. $m\widehat{ACD}$

27. $m\widehat{AG}$

28. $m\widehat{ACF}$

Inscribed Angles – Day 3

Warm – Up

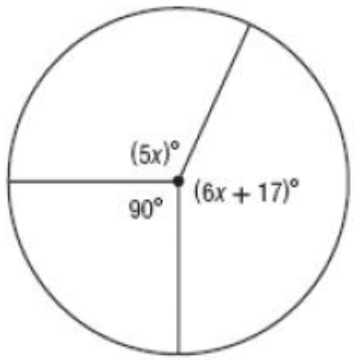
What is the value of x in the figure below?

(1) 21

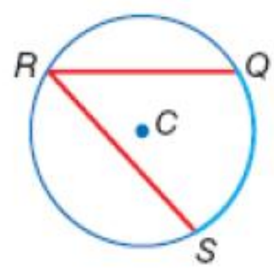
(2) 23

(3) 26

(4) 28

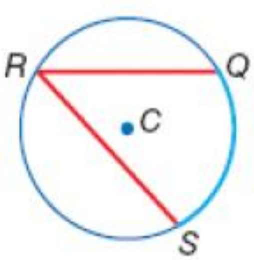


Inscribed Angles Notice that the angle formed by each streamer appears to be a right angle, no matter where point P is placed along the arch. An **inscribed angle** has a vertex on a circle and sides that contain chords of the circle. In $\odot C$, $\angle QRS$ is an inscribed angle.



An **intercepted arc** has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In $\odot C$, minor arc $\widehat{QS}$ is intercepted by $\angle QRS$.

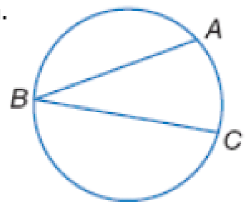
There are three ways that an angle can be inscribed in a circle.



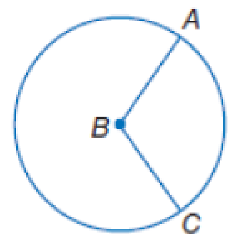
Model Problem #1

State if each angle is an inscribed angle. If it is, name the angle and the intercepted arc.

a.



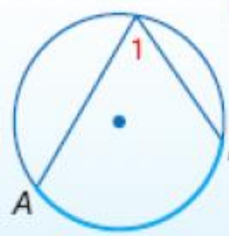
b.



Theorem 10.6 **Inscribed Angle Theorem** *For Your FOLDABLE*

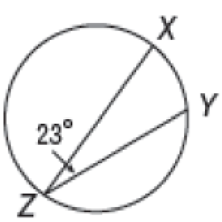
Words If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Example $m\angle 1 = \frac{1}{2}m\widehat{AB}$ and $m\widehat{AB} = 2m\angle 1$

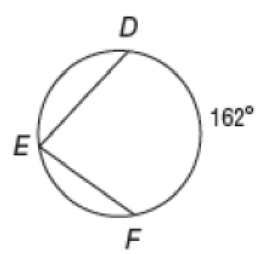


Model Problem #2

1. $m\widehat{XY}$

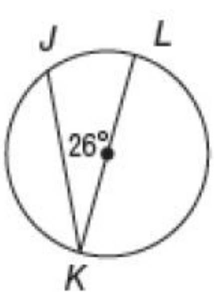


2. $m\angle E$

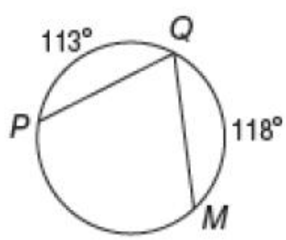


Check Your Progress

1. $m\widehat{JL}$



2. $m\angle Q$

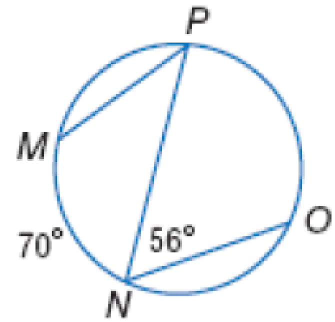


Level B – Problems

3. Find each measure.

a. $m\angle P$

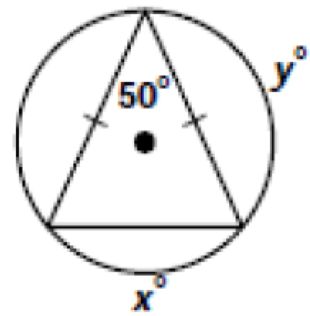
b. $m\widehat{PO}$



4. Find each measure.

a. m $\angle$x

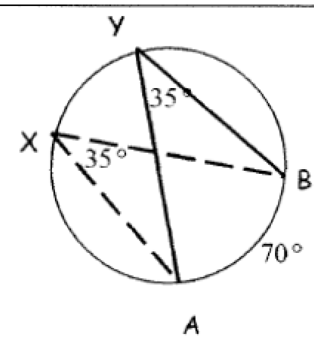
b. m $\angle$y



Thm: If two inscribed or tangent-chord angles intercept that same arc, then they are ≅.

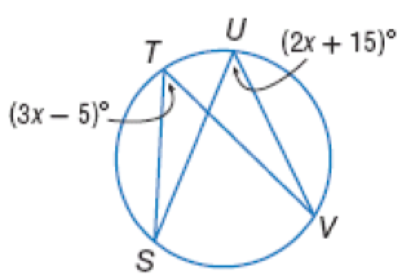
Given: X and Y are inscribed angles intercepting $\widehat{AB}$

Conc:______________

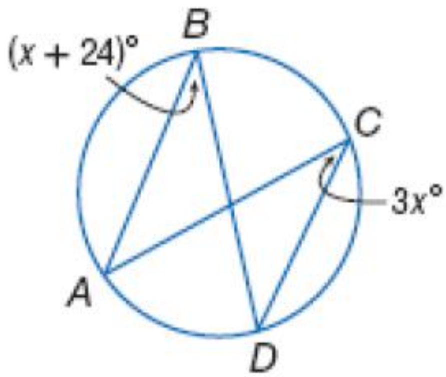


Model Problem #3

Find $m\angle T$.



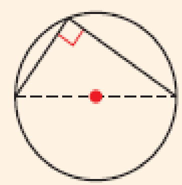
$m\angle B$



Angles of Inscribed Polygons Triangles and quadrilaterals that are inscribed in circles have special properties.

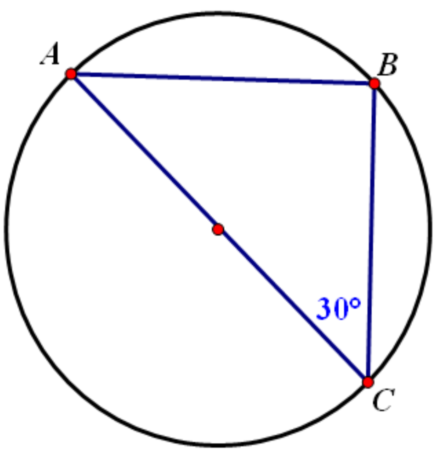
Theorem 11-4-3

An inscribed angle subtends a semicircle if and only if the angle is a right angle.



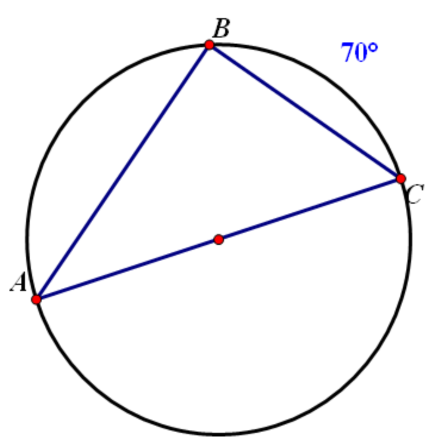
Model Problem #4

Find $m\widehat{BC}$



Model Problem #5

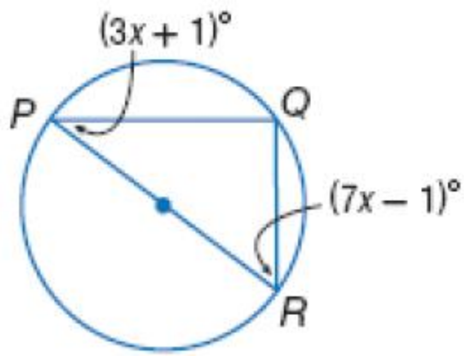
Find $m\measuredangle C$



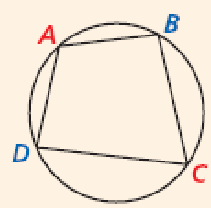
Level B

Check Your Progress

$m\angle R$



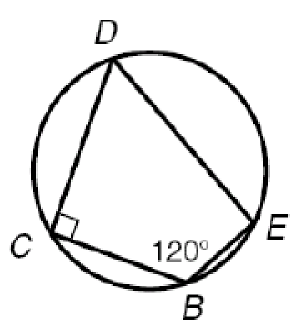
Theorem 11-4-4

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	 $ABCD$ is inscribed in $\odot E$.	$\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.

Model Problem #6

Find the angle measures of each inscribed quadrilateral.

a.



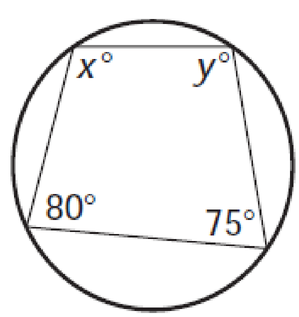
$m\angle B =$ ________

$m\angle C =$ ________

$m\angle D =$ ________

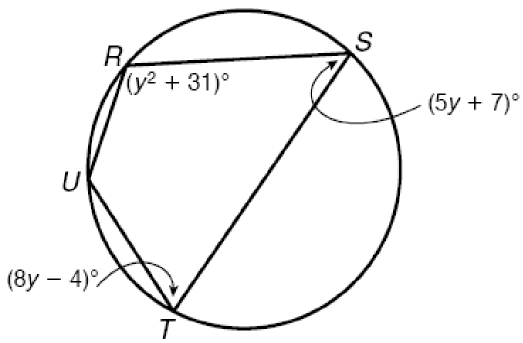
$m\angle E =$ ________

Check Your Progress



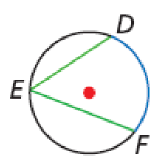
Challenge

Find the angle measures of *RSTU*.



SUMMARY

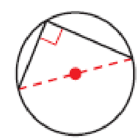
$\angle DEF$ is an inscribed angle.
$\overset{\frown}{DF}$ is an intercepted arc.
$\overset{\frown}{DF}$ *subtends* $\angle DEF$.



Inscribed Angle Theorem

The measure of an angle inscribed in a circle is half the measure of its intercepted arc. $m\angle DEF = \frac{1}{2} m\overset{\frown}{DF}$

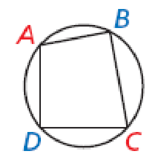
An inscribed angle subtends a semicircle if and only if the angle is a right angle.



A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

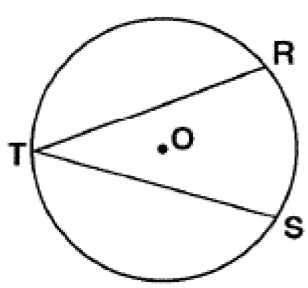
$\angle A$ and $\angle C$ are supplementary.

$\angle B$ and $\angle D$ are supplementary.



Exit Ticket

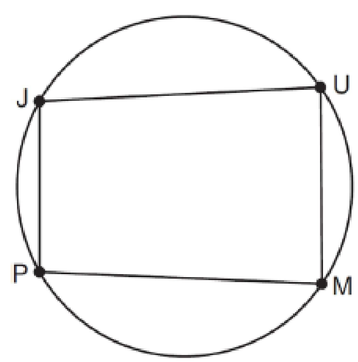
In the accompanying diagram of circle O, the measure of $\overset{\frown}{RS}$ is 64°.



What is m∠RTS?

A) 32°
B) 128°
C) 64°
D) 96°

2 In the diagram below, quadrilateral *JUMP* is inscribed in a circle..

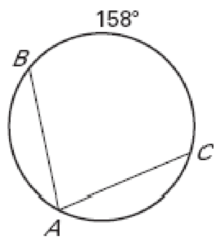


Opposite angles *J* and *M* must be
1) right
2) complementary
3) congruent
4) supplementary

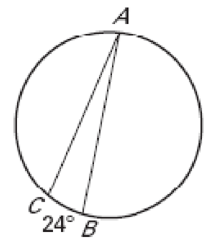
Day 3 – Homework

Find the indicated measure.

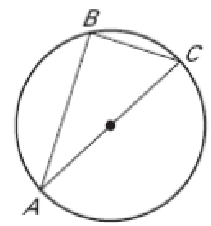
1) $m\angle A$



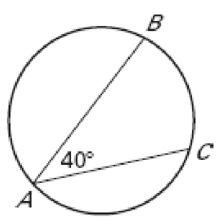
2) $m\angle A$



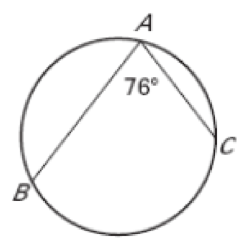
3) $m\angle B$



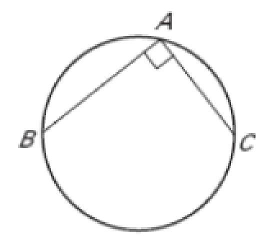
4) mBC



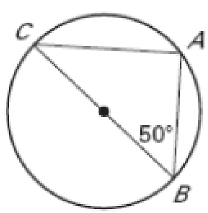
5) mBC



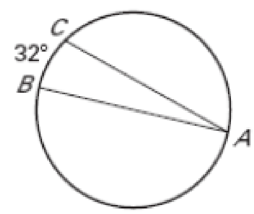
6) mBC



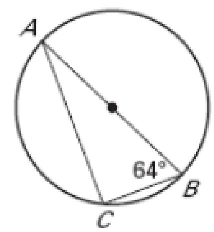
7) $m\angle C$



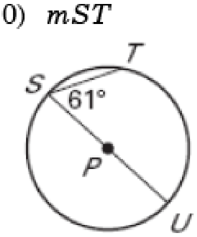
8) $m\angle A$



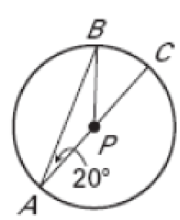
9) mBC



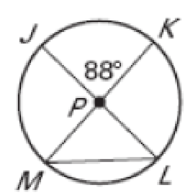
10) mST



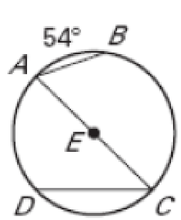
11) mAB



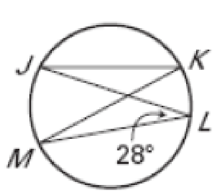
12) $m\angle JLM$



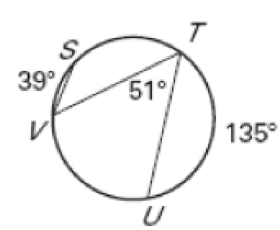
13) $m\angle A$



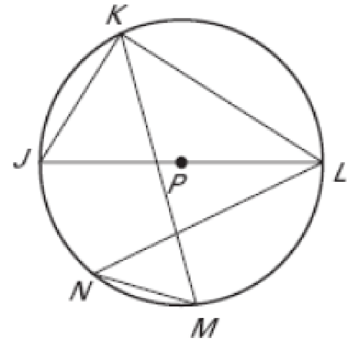
14) $m\angle K$



15) $mVST$



Find the measure of the indicated angle or arc in $\odot P$, given $mLM = 84^\circ$ and $mKN = 116^\circ$.



16) $m\angle JKL =$ _______

17) $m\angle MKL =$ _______

18) $m\angle KMN =$ _______

19) $m\angle JKM =$ _______

20) $m\angle KLN =$ _______

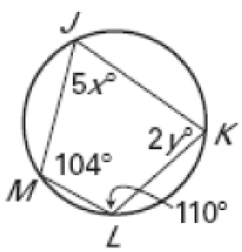
21) $m\angle LNM =$ _______

22) $mMJ =$ _______

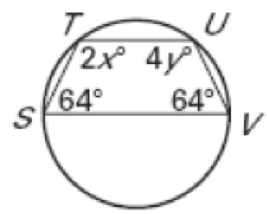
23) $mLKJ =$ _______

Find the value of the variables.

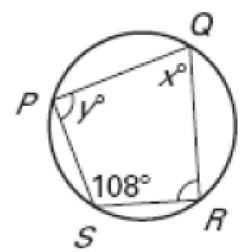
24)



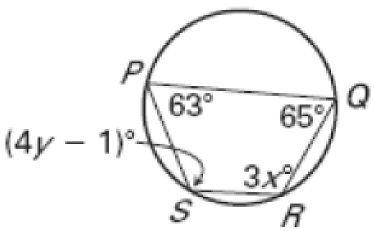
25)



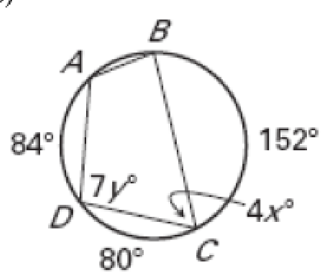
26)



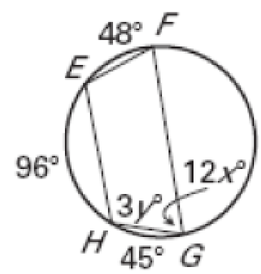
27)



28)



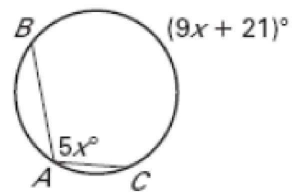
29)



30) What is the value of x in the figure shown?

A. 7 B. 12

C. 16 D. 21

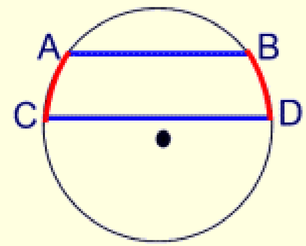


Day 4 – Review Day

Warm – Up

Theorem:

In a circle, parallel chords intercept congruent arcs.

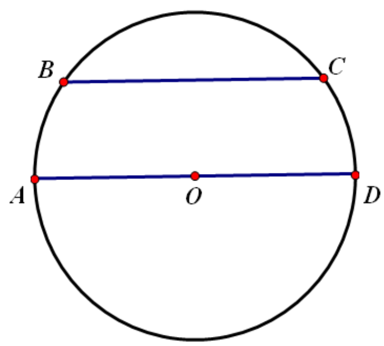


$\overline{AB} \parallel \overline{CD}; \quad \widehat{AC} \cong \widehat{BD}$

Example 1:

In the diagram of circle O below, chord $\overline{BC}$ is parallel to diameter $\overline{AOD}$ and m$\widehat{AB}$ = 30.

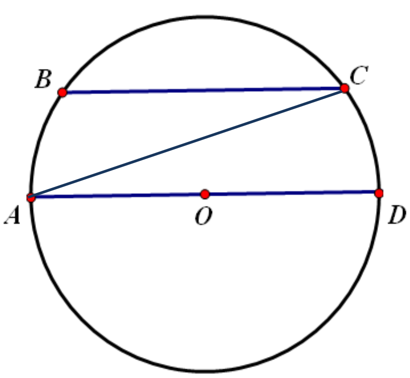
What is m$\widehat{BC}$?



Example 2:

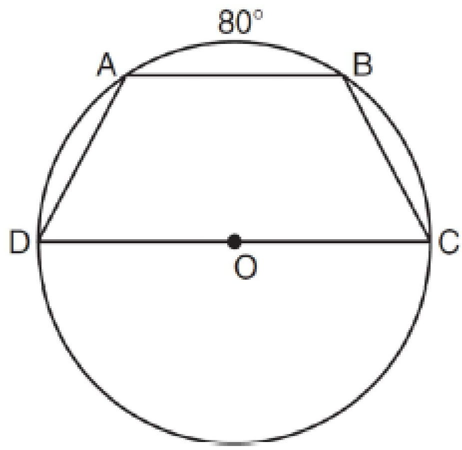
In the diagram of circle O below, chord $\overline{BC}$ is parallel to diameter $\overline{AOD}$ and m$\widehat{BC}$ = 100.

What is m $\measuredangle BCA$?

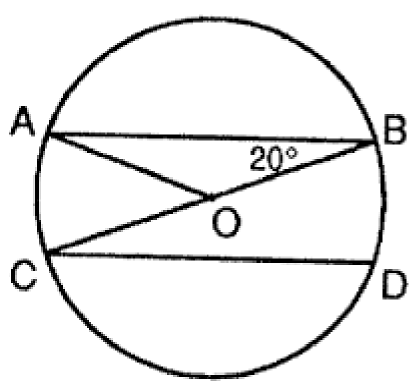


Practice

3. In the diagram below, trapezoid $ABCD$, with bases $\overline{AB}$ and $\overline{DC}$, is inscribed in circle O, with diameter $\overline{DC}$. If m$\overset{\frown}{AB}$=80, find m$\overset{\frown}{BC}$.

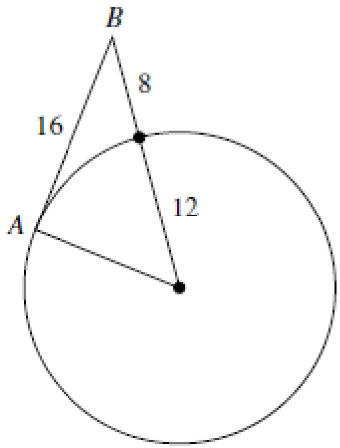


4. In the accompanying diagram of circle O, $\overline{AB} \parallel \overline{CD}$, $\overline{BC}$ is a diameter, and radius $\overline{AO}$ is drawn. If $m\angle ABC = 20$, find m$\overset{\frown}{BD}$.

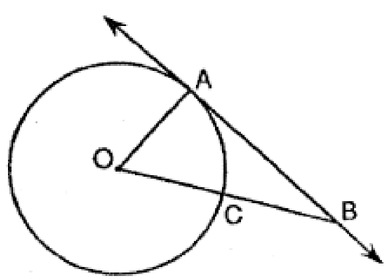


Section 1: Tangents of Circles

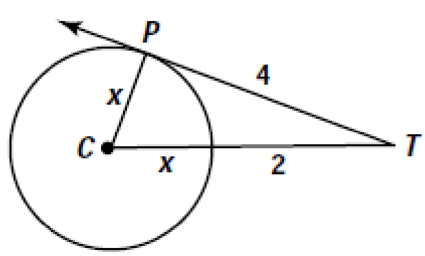
1. **Determine if line AB is tangent to the circle.**



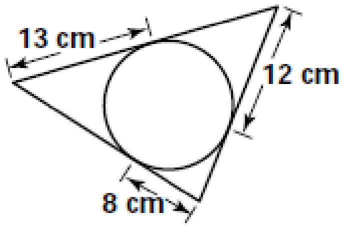
2. In the accompanying diagram, $\overleftrightarrow{BA}$ is tangent to circle O at A. Radii $\overline{OA}$ and $\overline{OC}$ are drawn, and $\overline{OC}$ is extended to intersect $\overleftrightarrow{BA}$ at B. If $BA = 15$ and $OB = 17$, find the measure of a radius of circle O.



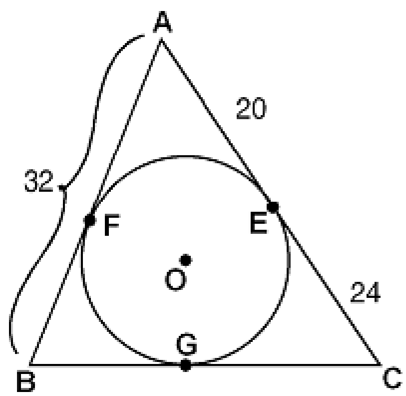
3. In each diagram, $\overline{TP}$ is tangent to $\odot C$ at P. Find the value of x.



4. **In each diagram, a polygon circumscribes a circle. Find the perimeter of each polygon.**

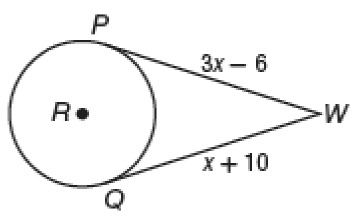


5. In the accompanying diagram, $\overline{AFB}$, $\overline{AEC}$, and $\overline{BGC}$ are tangent to circle O at F, E, and G, respectively.

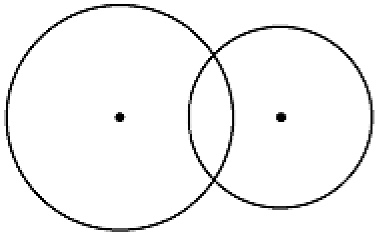


If AB = 32, AE = 20, and EC = 24, find BC.

6. **Find x. Assume that segments that appear to be tangent are tangent.**

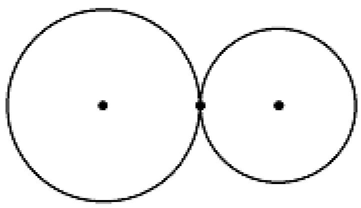


7. What is the number of common tangents that can be drawn to the circles below?



A) 1 B) 2 C) 3 D) 0

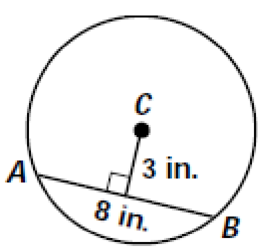
8. What is the number of common tangents that can be drawn to the circles below?



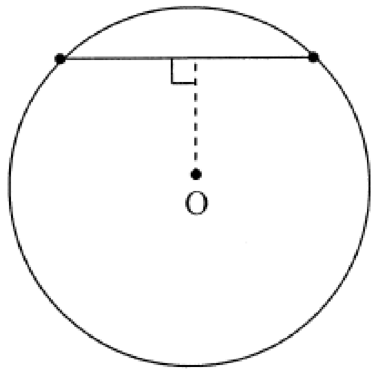
A) 1 B) 0 C) 3 D) 4

Section 2: Relating Arcs and Chords

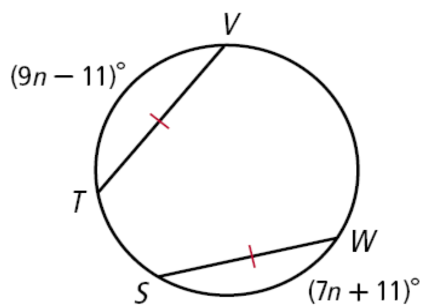
9. Calculate the radius.



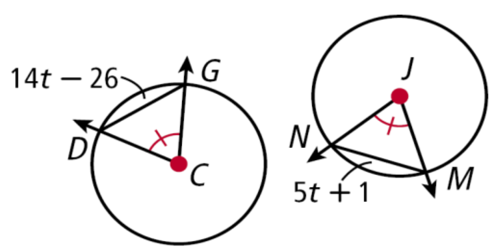
10. $\odot O$ with chord 9 units from the center and a radius of 41 units. Calculate the length of the chord.



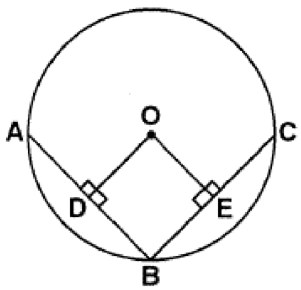
11. $\overline{TV} \cong \overline{WS}$. **Find $m\widehat{WS}$.**



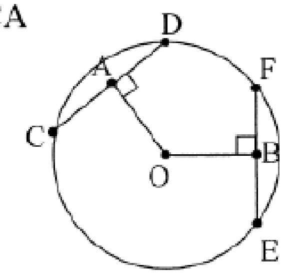
12. **$\odot C \cong \odot J$, and $m\angle GCD \cong m\angle NJM$. Find NM.**



In circle O below, $\overline{OD} \perp \overline{AB}$, $\overline{OE} \perp \overline{BC}$, and $\overline{OD} \cong \overline{OE}$.

13. 

13b. Given: $\odot O$, $\overline{OA} \cong \overline{OB}$, $CD = 2x + 4$, $BF = 2x - 1$.
Find: CA



If EC = 6, find AB.

If EC = 5, find AB.

If AB = 14, find EC.

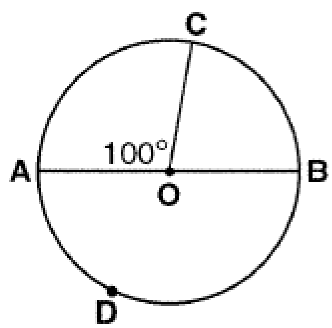
Section 3: Central Angles

14. In circle O below, $\overline{AB}$ is a diameter and $m\angle AOC = 100°$.

Find the value of the following:

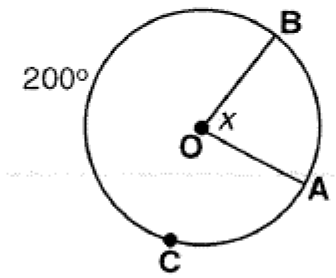
$m\widehat{AC}$

$m\widehat{ADC}$

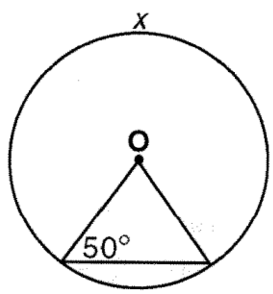


15. In the accompanying diagram of circle O, arc ACB has a measure of 280°.

What is the measure of central angle x?



16. For the given circle, find the value of x.



17. **Find each measure.**

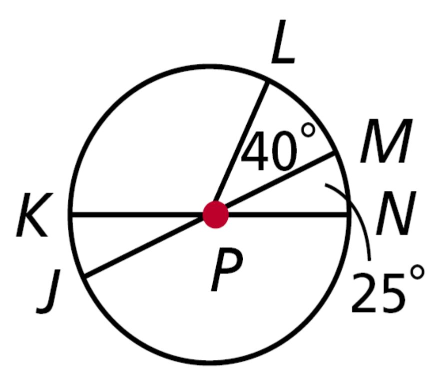
a. **m** $\widehat{MN}$

b. **m** $\widehat{KL}$

c. **m** $\widehat{KJ}$

d. **m** $\widehat{JN}$

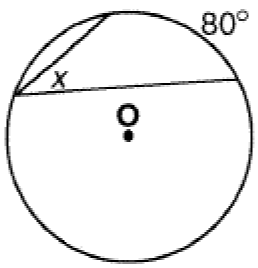
e. **m** $\widehat{JLM}$



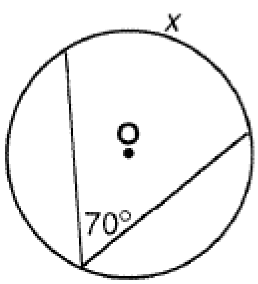
Section 4: Inscribed Angles

For the given circle, find the value of x.

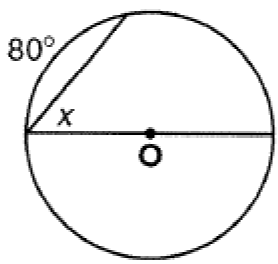
18.



19.



20.



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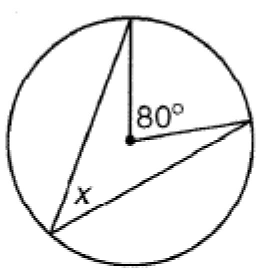
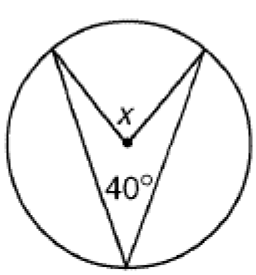
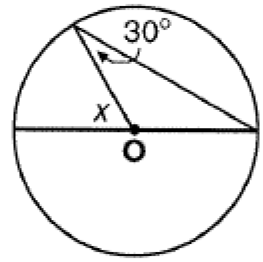


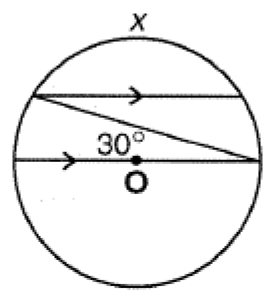
22.



23.

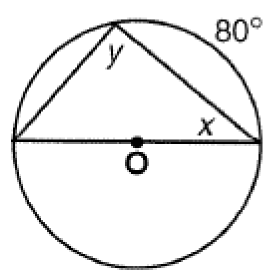


24.

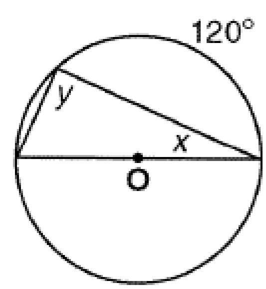


For the given circle, find the value of x and y.

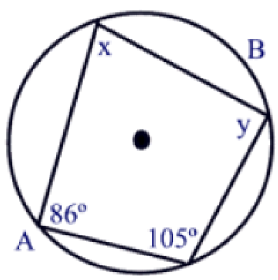
25.



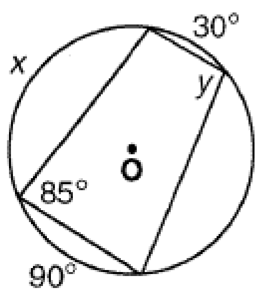
26.



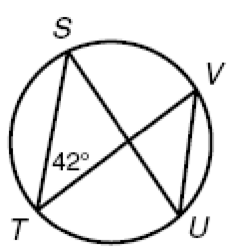
27. For the given circle, find the value of x and y.



28. For the given circle, find the value of x and y.

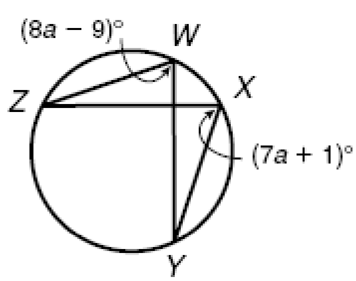


29.



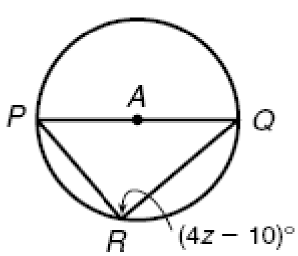
$m\angle VUS =$ __________

30.



$m\angle ZWY =$ __________

31.



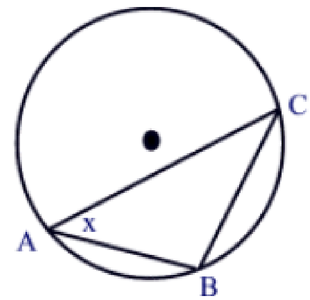
$z =$ __________

Day 5 – Angle Relationships in Circles

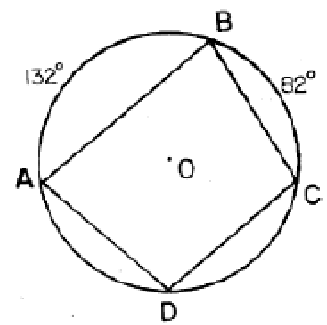
Warm - Up

1. Given circle with center indicated and $m\overset{\frown}{AB} : m\overset{\frown}{BC} : m\overset{\frown}{AC} = 3:4:8$

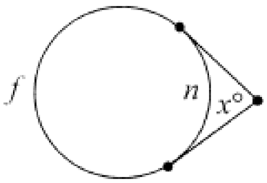
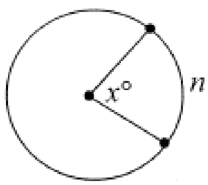
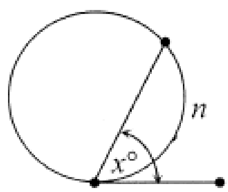
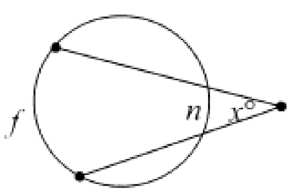
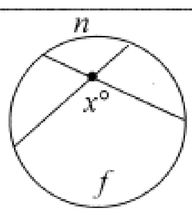
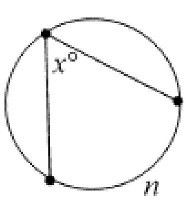
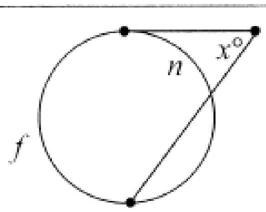
Find x.



2. In the accompanying diagram, quadrilateral $ABCD$ is inscribed in circle O. If $m\overset{\frown}{AB} = 132$ and $m\overset{\frown}{BC} = 82$, find $m\angle ADC$.

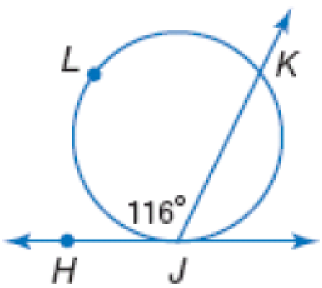


Next to each diagram, give the name of the angle shown and write down the formula used to find its measure.

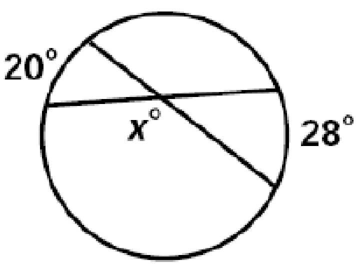
Diagram	Location of Vertex	Classification	Formula
1. 			
2. 			
3. 			
4. 			
5. 			
6. 			
7. 			

Example 1: Vertex is _____ circle

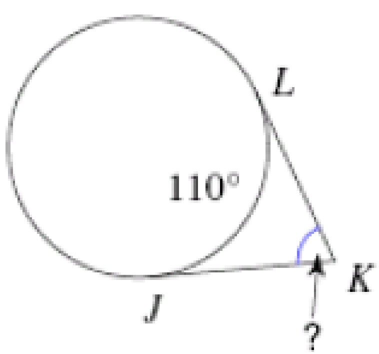
Find $m\widehat{JLK}$.



Example 2: Vertex is __________ circle



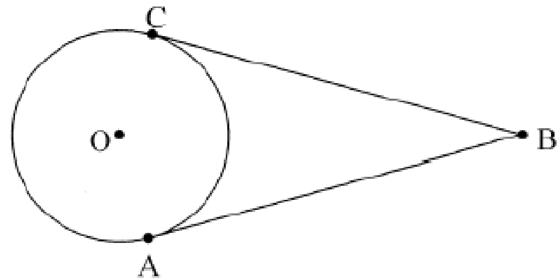
Example 3: Vertex is __________ circle



Theorem: The sum of the measures of a tangent-tangent angle and its minor arc is 180.

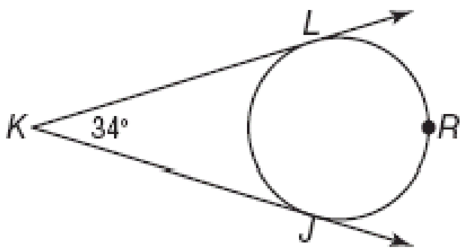
Given: $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$

Conclusion: $m\angle B + m\widehat{AC} = 180$

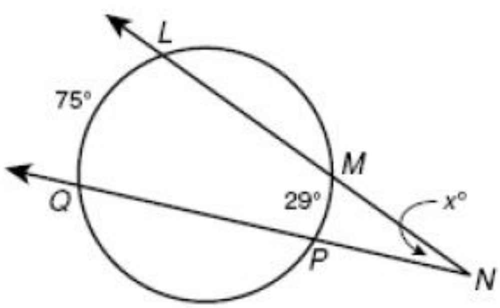


Example 4:

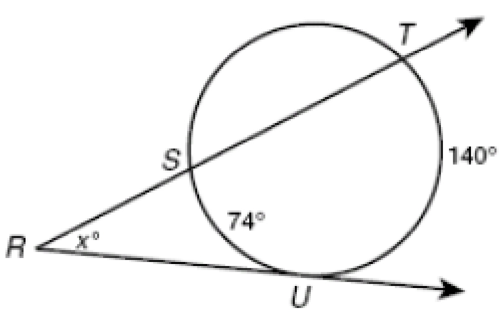
$m\widehat{LJ}$



Example 5: Vertex is __________ circle



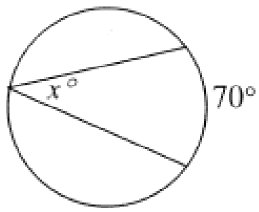
Example 6: Vertex is __________ circle



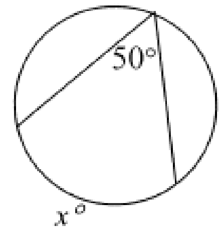
Key Question: Where's the vertex?

For each question, write out the appropriate formula, plug in the known values, then solve.

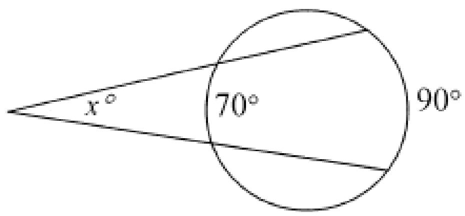
1.



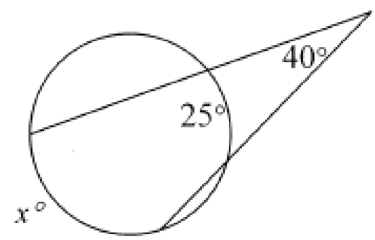
2.



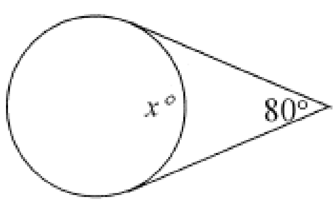
3.



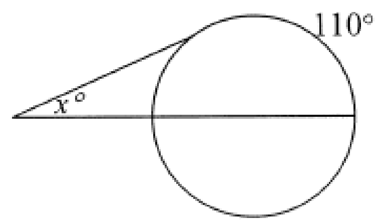
4.



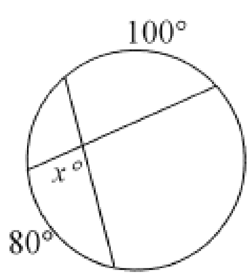
5.



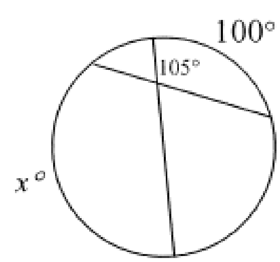
6.



7.

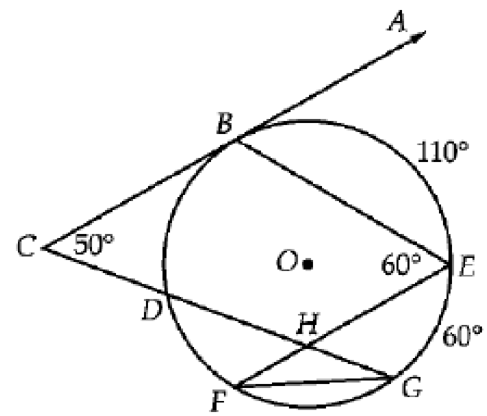


8.



CHALLENGE

Given: Circle O with tangent $\overrightarrow{CBA}$, secant $\overline{CDG}$.
Chords $\overline{BE}$, $\overline{FG}$, and $\overline{EF}$. $m\angle BEF = 60$,
$m\widehat{BE} = 110$, $m\widehat{GE} = 60$, $m\angle BCG = 50$.

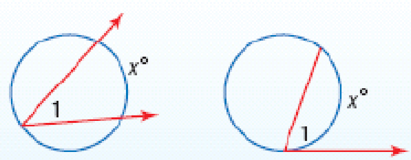
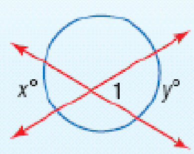
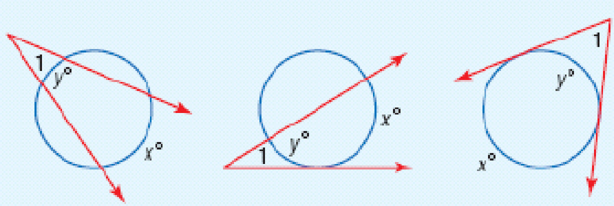


Find:

a. $m\widehat{BDF}$
b. $m\widehat{FG}$
c. $m\widehat{BD}$
d. $m\angle DGF$
e. $m\angle CHE$
f. $m\angle ABE$

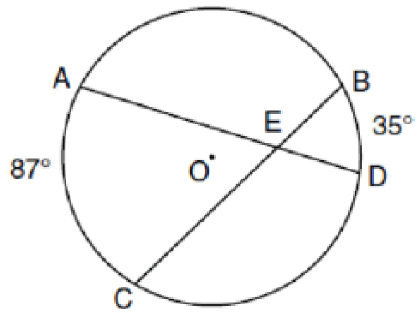
Summary

Concept Summary — Circle and Angle Relationships — For Your FOLDABLE

Vertex of Angle	Model(s)	Angle Measure
on the circle $2(m\measuredangle 1) = x$		one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$
inside the circle $2(m\measuredangle 1) = x + y$		one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$
outside the circle $2(m\measuredangle 1) = x - y$		one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$

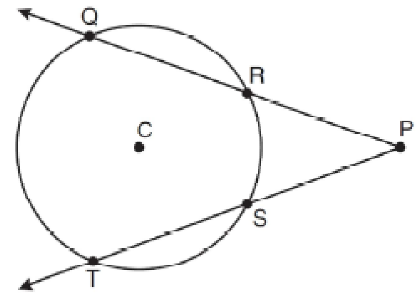
Exit Ticket

1 In the diagram below of circle O, chords $\overline{AD}$ and $\overline{BC}$ intersect at E, $m\overset{\frown}{AC} = 87$, and $m\overset{\frown}{BD} = 35$.



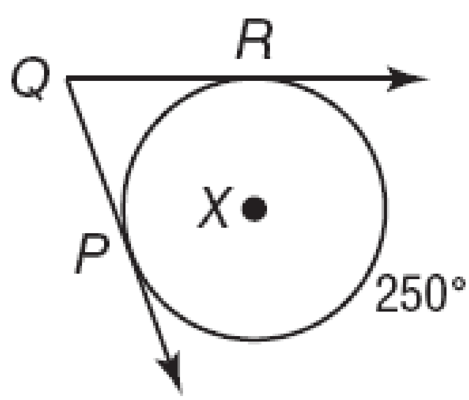
What is the degree measure of $\angle CEA$?
1) 87
2) 61
3) 43.5
4) 26

2. In the diagram below of circle C, $m\overset{\frown}{QT} = 140$, and $m\angle P = 40$.



What is $m\overset{\frown}{RS}$?
1) 50
2) 60
3) 90
4) 110

3. Find $m\angle PQR$ if $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are tangent to $\odot X$.

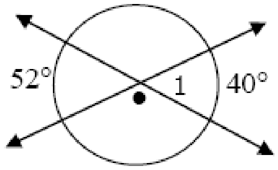


A 70°
B 110°
C 125°
D 140°

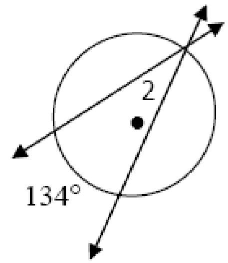
Homework

Find the measure of each numbered angle.

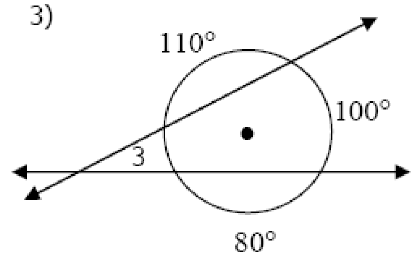
1)



2)

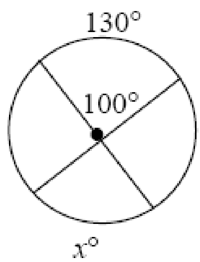


3)

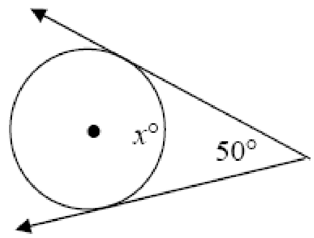


Find the value of x.

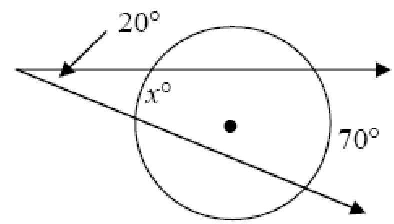
4)



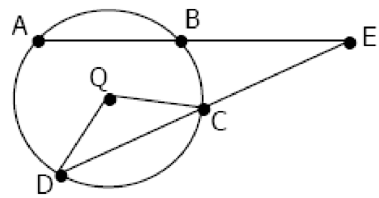
5)



6)



Assume that lines that appear to be tangents are tangents. In $\odot Q$, $m\angle CQD = 120°$, $m\widehat{BC} = 30°$, and $m\angle BEC = 25°$. Find each measure.



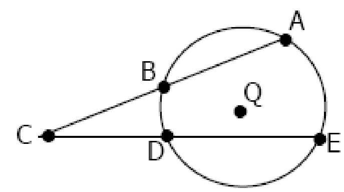
7) $m\widehat{DC}$

8) $m\widehat{AD}$

9) $m\widehat{AB}$

10) $m\angle QDC$

In $\odot Q$, $m\widehat{AE} = 140°$, $m\widehat{BD} = y°$, $m\widehat{AB} = 2y°$, and $m\widehat{DE} = 2y°$. Find each measure.



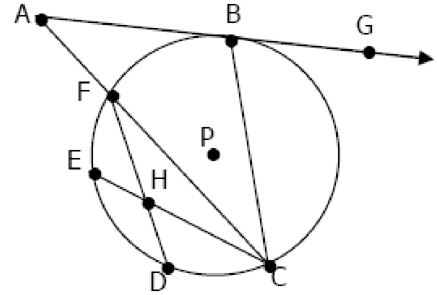
11) $m\widehat{BD}$

12) $m\widehat{AB}$

13) $m\widehat{DE}$

14) $m\angle BCD$

In $\odot P$, $m\widehat{BC} = 4x - 50$, $m\widehat{DE} = x + 25$, $m\widehat{EF} = x - 15$, $m\widehat{FB} = 50$, and $m\widehat{CD} = x$. Find each measure.



15) $m\angle A$

16) $m\angle BCA$

17) $m\angle ABC$

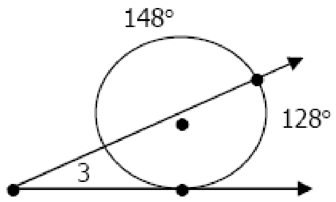
18) $m\angle GBC$

19) $m\angle FHE$

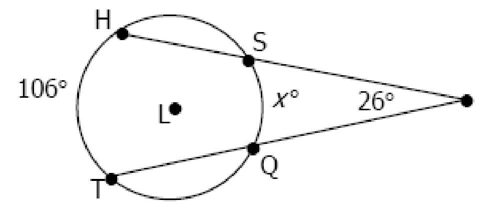
20) $m\angle CFD$

Use the diagram to find the missing information.

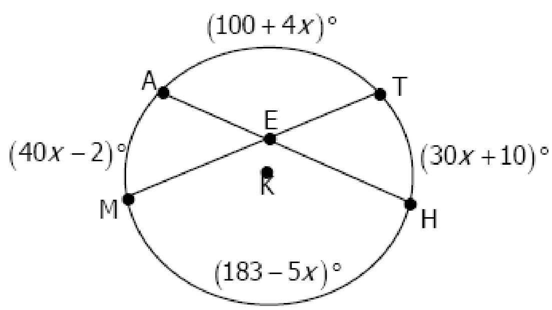
21) Find $m\measuredangle 3$



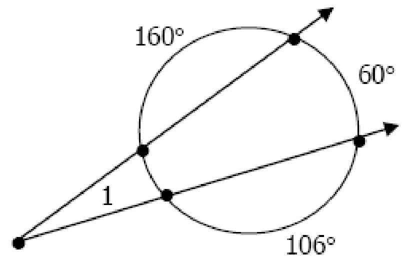
22) Find the value of x.



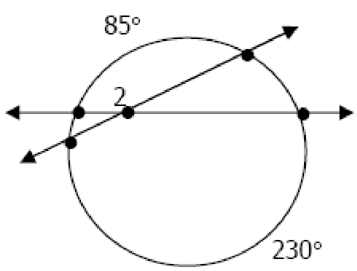
23) Find the value of x and $m\measuredangle AET$.



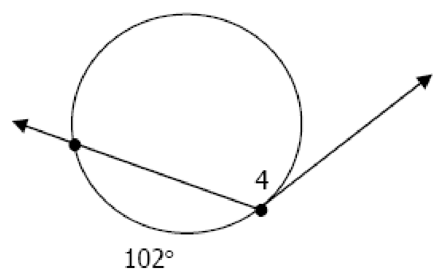
24) Find $m\measuredangle 1$.



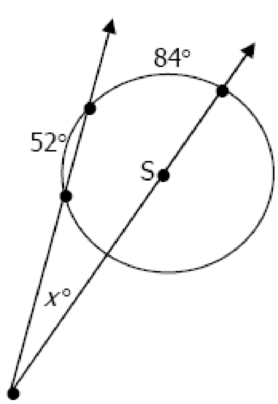
25) Find $m\measuredangle 2$,



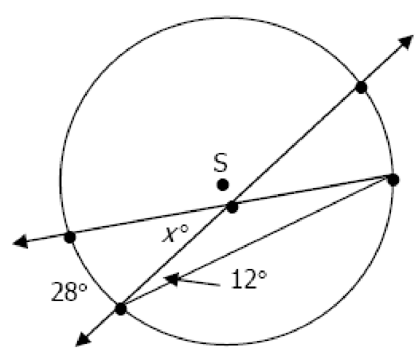
26) Find $m\measuredangle 4$.



27) Find the value of x.



28) Find the value of x.



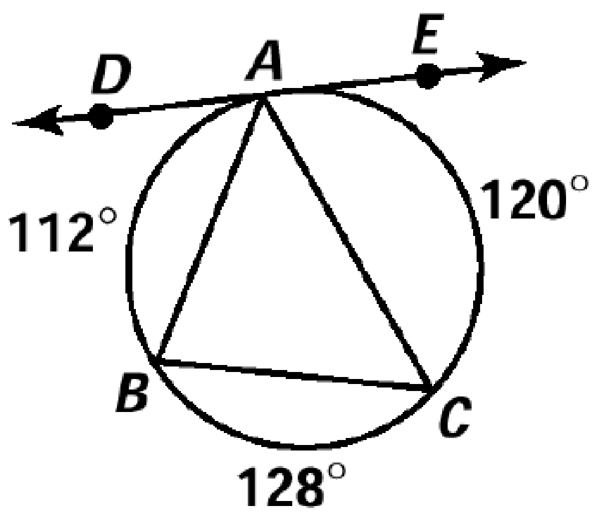
Answers

1. 46
2. 67
3. 15
4. 70
5. 130
6. 30
7. 120
8. 80
9. 130
10. 30
11. 44
12. 88
13. 88
14. 48
15. 50
16. 25
17. 105
18. 75
19. 42.5
20. 25
21. 22
22. 54
23. 1, 141
24. 13
25. 157.5
26. 129
27. 20
28. 26

Day 6 – More with Angle Relationships in Circles

Warm – Up

1. **Find each measure.**



a. m $\angle BAC$

b. m$\angle ABC$

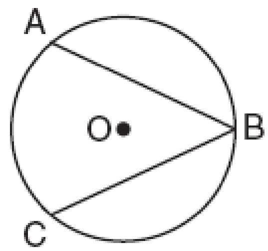
c. m$\angle BCA$

d. m$\angle DAB$

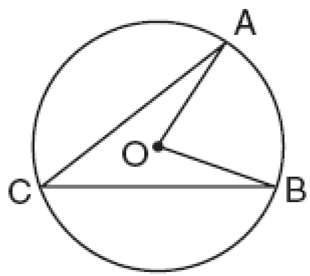
e. m$\angle EAC$

Review of Angles in Circles:

1. In the accompanying diagram of circle O, $m\widehat{ABC} = 260$. What is $m\angle ABC$?



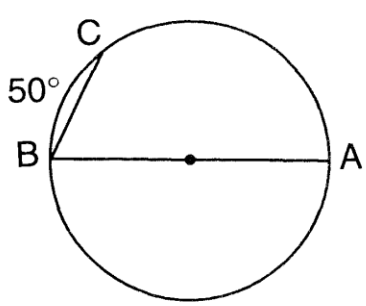
2. In the accompanying diagram of circle O, $m\angle ACB = 38$.



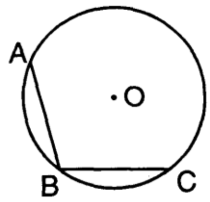
What is $m\angle AOB$?

(1) 19 (3) 52
(2) 38 (4) 76

3. In the accompanying diagram, $\overline{BA}$ is a diameter and $m\widehat{BC} = 50$. Find $m\angle CBA$.



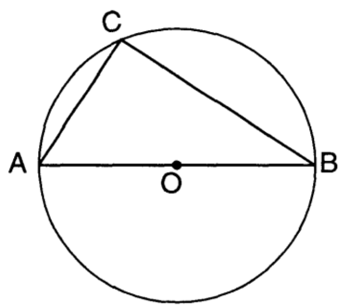
4. In the accompanying diagram of circle O, $m\overset{\frown}{ABC} = 150$.



What is $m\angle ABC$?

(1) 210 (3) 95
(2) 105 (4) 75

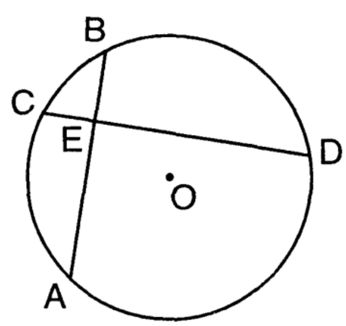
5. In the accompanying diagram, $\triangle ABC$ is inscribed in circle O and $\overline{AB}$ is a diameter.



What is the number of degrees in $m\angle C$?

(1) 30 (3) 60
(2) 45 (4) 90

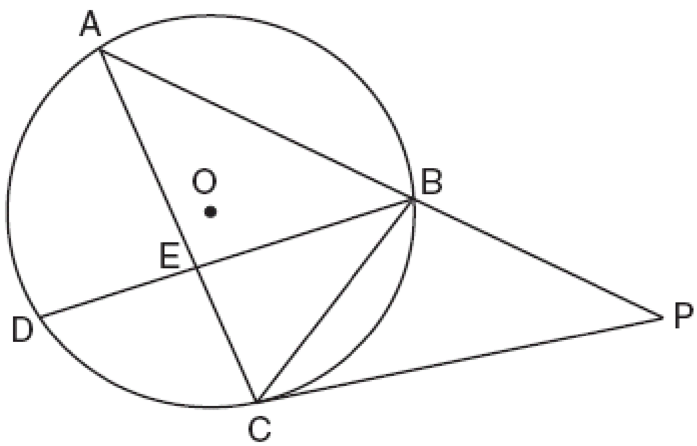
6. In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E and $m\overset{\frown}{AC}:m\overset{\frown}{CB}:m\overset{\frown}{BD}:m\overset{\frown}{DA} = 4:2:6:8$.



What is $m\angle DEB$?

(1) 36 (3) 100
(2) 90 (4) 126

7. In the accompanying diagram of circle O, chords $\overline{BD}$, $\overline{BC}$, and $\overline{AC}$, tangent $\overline{PC}$, and secant $\overline{ABP}$ are drawn; $m\angle DBC = 40$; $m\angle AEB = 110$; and $m\widehat{AD}:m\widehat{CB} = 9:5$.



Find:

a $m\widehat{AB}$ [2]

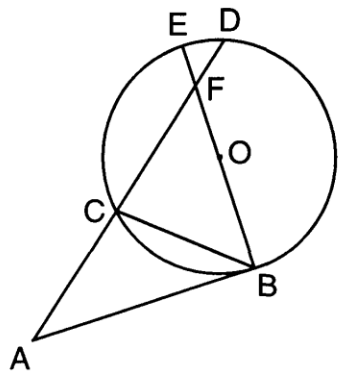
b $m\widehat{AD}$ [2]

c $m\angle P$ [2]

d $m\angle BCP$ [2]

e $m\angle ACP$ [2]

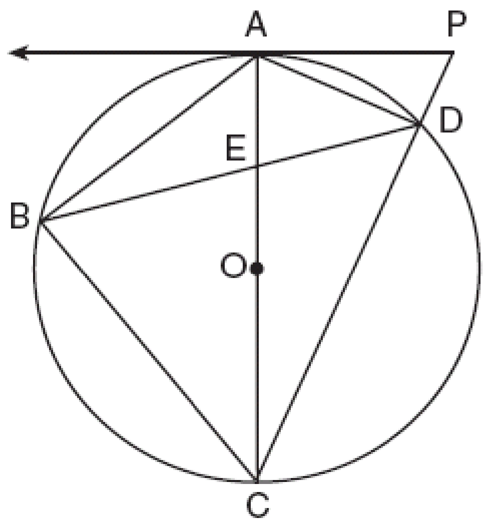
8. In the accompanying diagram of circle O, tangent $\overline{AB}$ and chord $\overline{BC}$ are drawn, secant $\overline{ACD}$ intersects diameter $\overline{EB}$ at F, $m\widehat{BD} = 160$, and $m\widehat{BC} = 80$.



Find:

a $m\angle A$ [2]
b $m\angle ABE$ [2]
c $m\angle ABC$ [2]
d $m\angle EFC$ [2]
e $m\angle ACB$ [2]

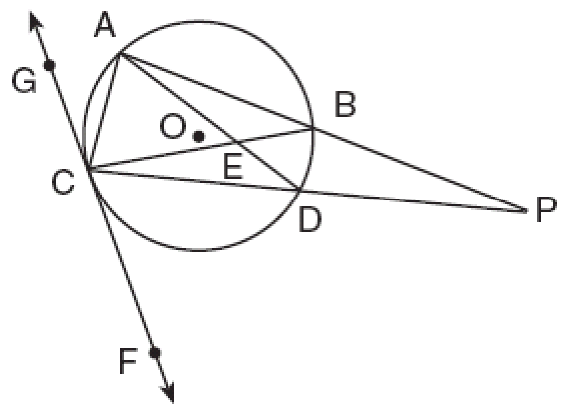
9. In the accompanying diagram of circle O, $\overrightarrow{PA}$ is tangent to the circle at A; $\overline{PDC}$ is a secant; diameter $\overline{AEOC}$ intersects chord $\overline{BD}$ at E; chords $\overline{AB}$, $\overline{BC}$, and $\overline{DA}$ are drawn; m$\widehat{DA}$ = 46; and m$\widehat{BC}$ is 32 more than m$\widehat{AB}$.



Find:

a m$\widehat{AB}$ [2]
b m$\angle BAC$ [2]
c m$\angle P$ [2]
d m$\angle DEC$ [2]
e m$\angle PDA$ [2]

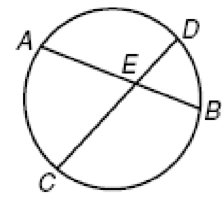
10. In the accompanying diagram of circle O secant $\overline{ABP}$, secant $\overline{CDP}$, and chord $\overline{AC}$ are drawn; chords $\overline{AD}$ and $\overline{BC}$ intersect at E tangent $\overleftrightarrow{GCF}$ intersects circle O at C, and $m\overset{\frown}{AB}:m\overset{\frown}{BD}:m\overset{\frown}{DC}:m\overset{\frown}{CA} = 8:2:5:3$.



Find:

a $m\overset{\frown}{CA}$ [2]
b $m\angle ACB$ [2]
c $m\angle P$ [2]
d $m\angle AEB$ [2]
e $m\angle DCF$ [2]

Day 7 – Segment Relationships of Circles

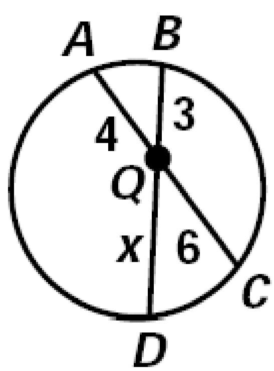
Chord-Chord Product Theorem	
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.	 $AE \cdot EB = CE \cdot ED$

Level A

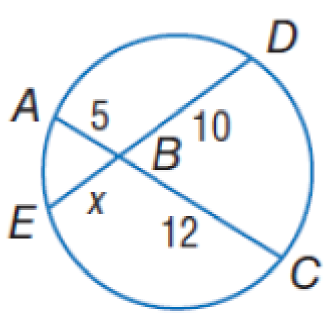
Model Problem #1

Find the value of x.

1.

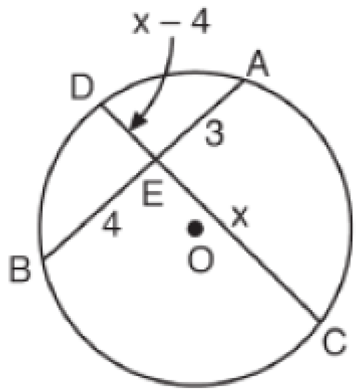


Check Your Progress



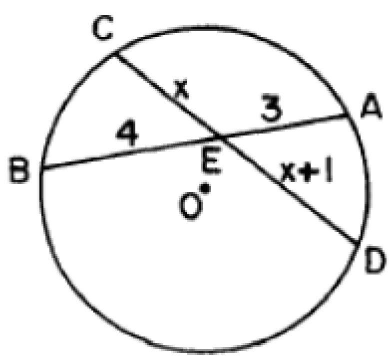
Level B

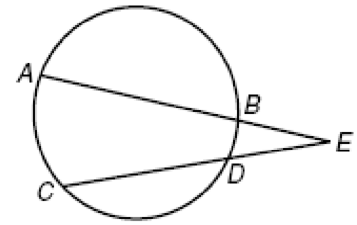
In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x - 4$, what is the value of x?



Check Your Progress

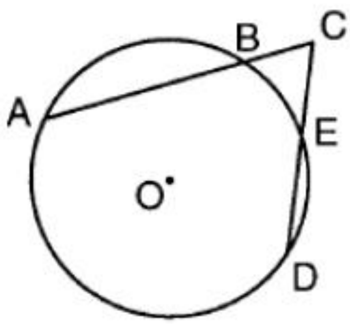
In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x + 1$, find CE.



Secant-Secant Product Theorem The product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. whole • outside = whole • outside $AE \cdot BE = CE \cdot DE$	

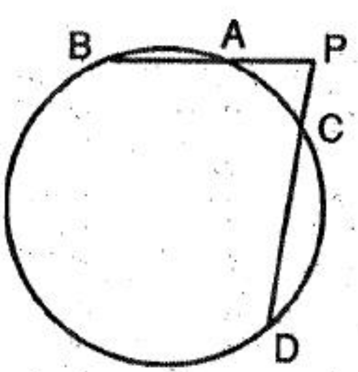
Find the value of x.

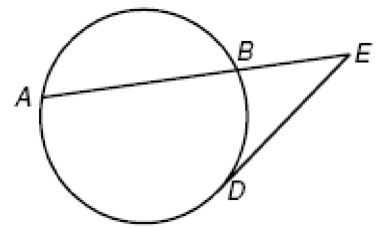
3. In the accompanying diagram of circle O, secants $\overline{CBA}$ and $\overline{CED}$ intersect at C. If $AC = 12$, $BC = 3$, and $DC = 9$, find EC.



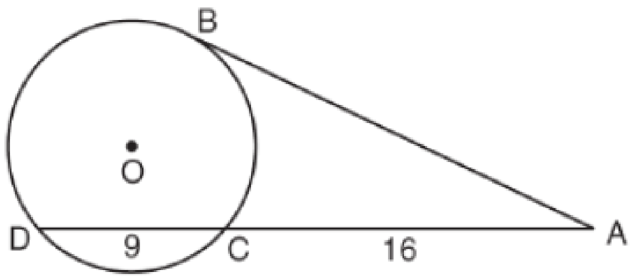
4. Check Your Progress

In the diagram below, $\overline{PAB}$ and $\overline{PCD}$ are secants to the circle. If $PA = 4$, $AB = 5$, and $PD = 12$, what is PC?



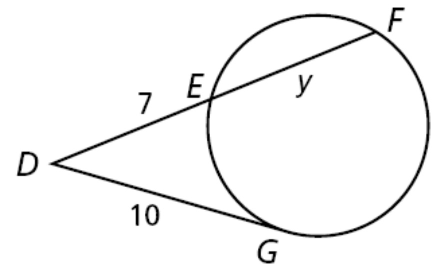
Secant-Tangent Product Theorem The product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. whole • outside = tangent^2 $AE \cdot BE = DE^2$	

5. In the accompanying diagram $\overline{AB}$, is tangent to circle O at B. If $AC = 16$ and $CD = 9$, what is the length of $\overline{AB}$?



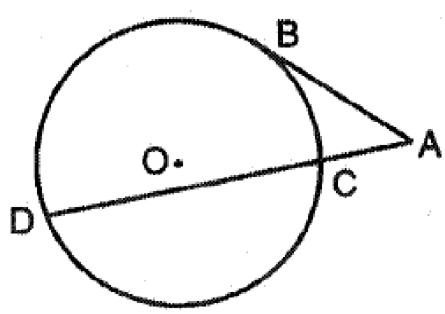
Find the value of *y*.

6.

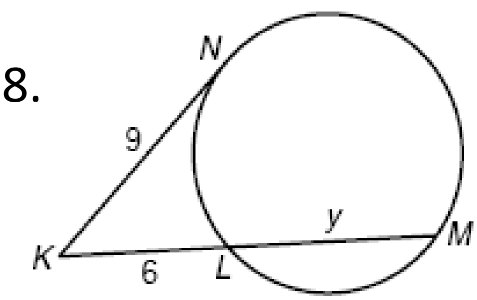


Check Your Progress

7. In the accompanying diagram, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle O from point A. If $AC = 4$ and $CD = 12$, find AB.

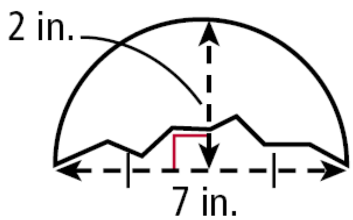


8.



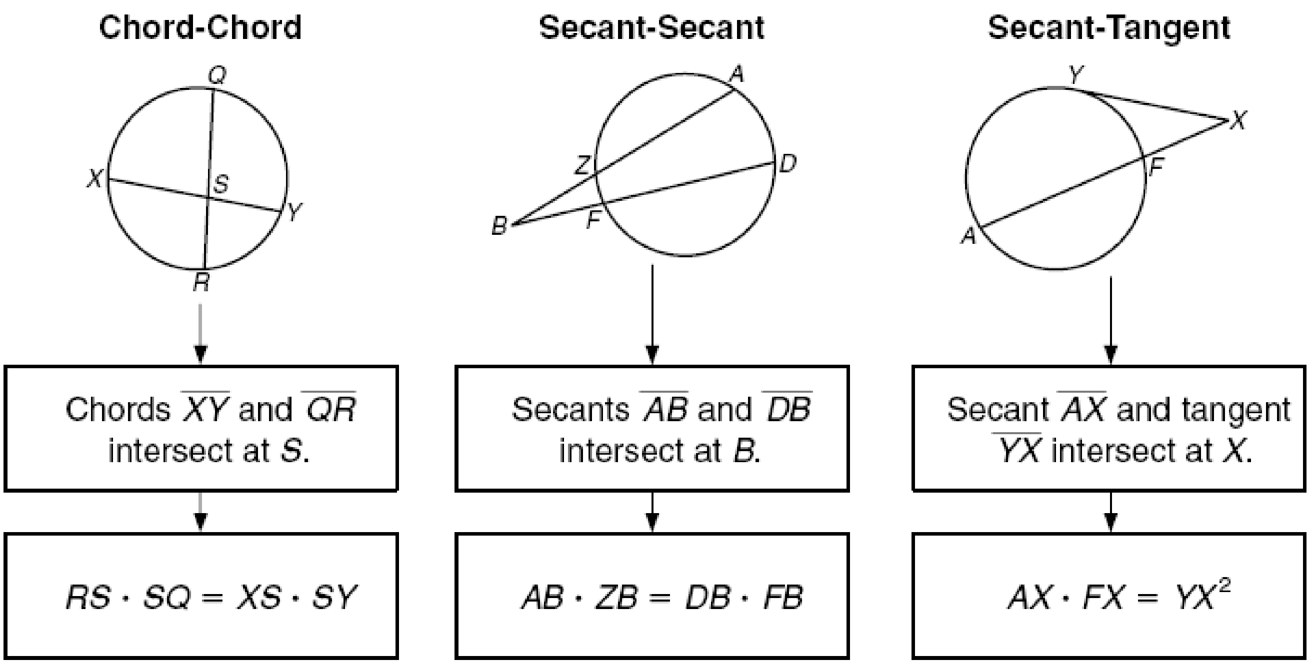
Challenge

Find the diameter of the plate.



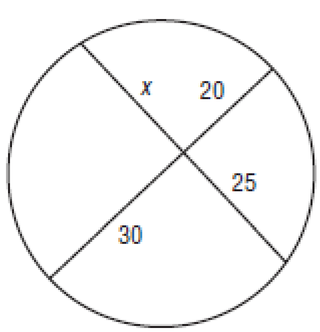
Summary

The models below show segment relationships in circles.



Exit Ticket

Solve for x in the figure below.



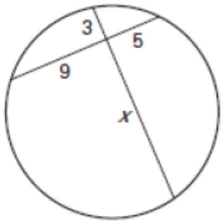
(1) 15 (3) 23

(2) 20 (4) 24

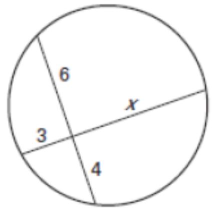
Homework

Fill in the blanks. Then find the value of x.

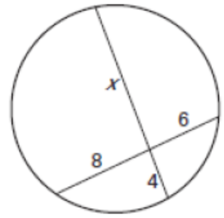
1) $x \cdot$ ________ $= 5 \cdot$ ________



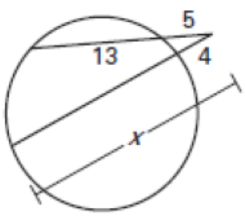
2) $6 \cdot$ ________ $= 3 \cdot$ ________



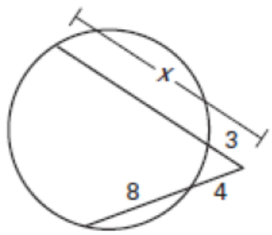
3) $x \cdot$ ________ $= 8 \cdot$ ________



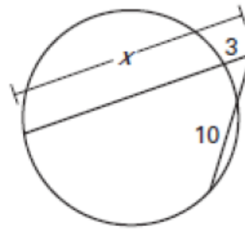
4) $4 \cdot$ ________ $= 5 \cdot$ ________



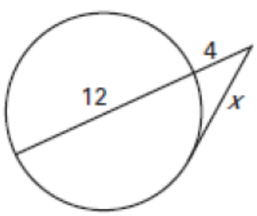
5) $3 \cdot$ ________ $= 4 \cdot$ ________



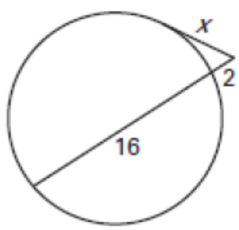
6) $3 \cdot$ ________ $= 5 \cdot$ ________



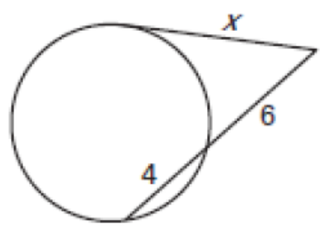
7) $x^2 = 4 \cdot$ ________



8) $x^2 = 2 \cdot$ ________

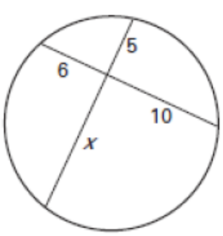


9) $x^2 = 6 \cdot$ ________

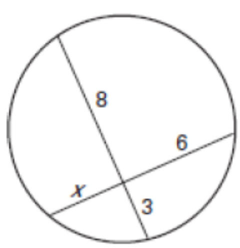


Find the value of x.

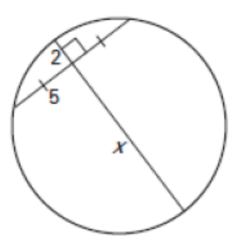
10) $x =$ ________



11) $x =$ ________

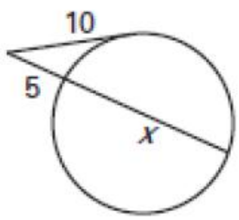


12) $x =$ ________

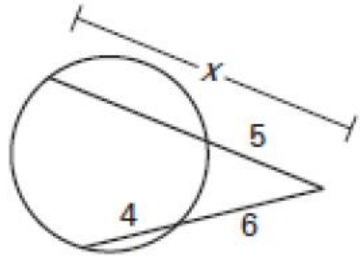


Find the value of x.

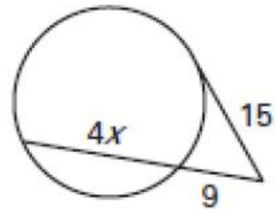
13) $x =$ _______



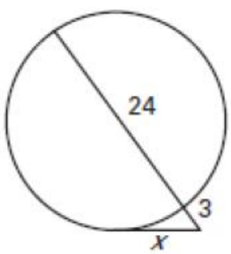
14) $x =$ _______



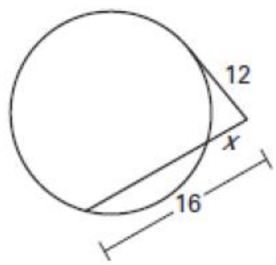
15) $x =$ _______



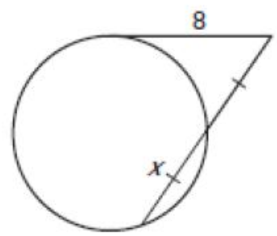
16) $x =$ _______



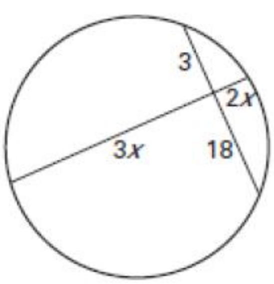
17) $x =$ _______



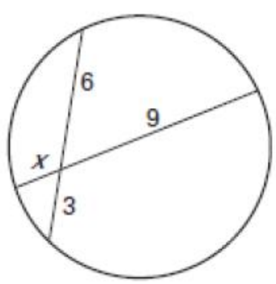
18) $x =$ _______



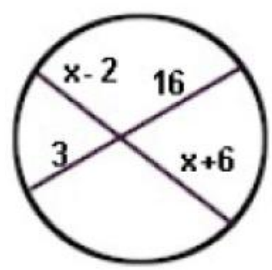
19) $x =$ _______



20) $x =$ _______



21) $x =$ _______



Day 8 & 9 - REVIEW

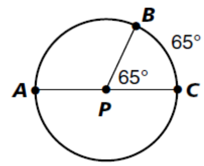
Arcs, Central Angles, and Inscribed Angles

Remember

1. An angle whose vertex is the center of a circle is a **central angle**. Example: $\angle BPC$
2. An **arc** is a curve of a circle. It is named by its endpoints.

   A **minor arc** measures less than 180°. Its measure is equal to the measure of its central angle. Example: $m\widehat{BC} = m\angle BPC = 65°$

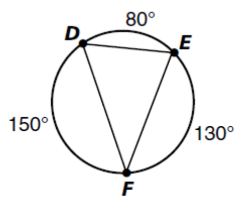
   A **semicircle** measures 180°. Its central angle is a diameter. Example: $m\widehat{AC} = m\angle APC = 180°$



A whole circle measures 360°.

   A **major arc** measures more than 180°. Its measure is the difference between 360° and the measure of its central angle. Example: $m\widehat{BAC} = 360° - 65° = 295°$

3. A **chord** is a segment whose endpoints are points on a circle. Example: $\overline{DF}$
   An **inscribed angle** is an angle whose sides are chords and whose vertex is a point on the circle. Example: $\angle DFE$



4. The measure of an inscribed angle is equal to half the measure of its intercepted arc.

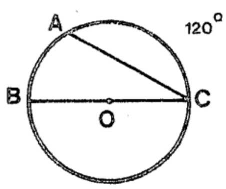
$m\angle DFE = \frac{1}{2}\, m\widehat{DE} = \frac{1}{2}(80°) = 40°$

$m\angle EDF = \frac{1}{2}\, m\widehat{EF} = \frac{1}{2}(130°) = 65°$

$m\angle DEF = \frac{1}{2}\, m\widehat{DF} = \frac{1}{2}(150°) = 75°$

Note: The sum of the angles of $\triangle DEF$ = 180°.

1. In the accompanying diagram, $\overline{BC}$ is a diameter and $\widehat{AC} = 120°$. How many degrees are there in $m\angle ACB$?



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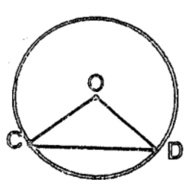
2. In the accompanying figure of circle O, radii OC and OD are drawn. If $m\angle D = 44°$, the $m\widehat{CD}$ is
   (1) 92° (3) 88°
   (2) 46° (4) 44°



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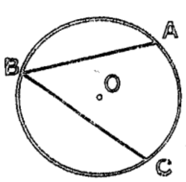
3. In the diagram below, find the measure of $\angle ABC$ if the $m\widehat{AC} = 92°$.
   (1) 92° (3) 23°
   (2) 184° (4) 46°



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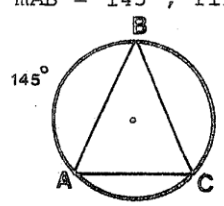
4. In the accompanying diagram, isosceles $\triangle ABC$ is inscribed in the circle. If $\overline{AB} \cong \overline{CB}$ and $m\widehat{AB} = 145°$, find $m\angle B$.



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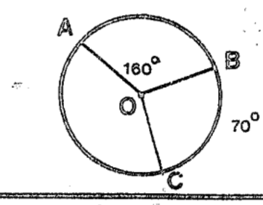
5. In the diagram of circle O, radii OA, OB, and OC are drawn. If $m\angle AOB = 160°$ and $m\widehat{BC} = 70°$, find $m\angle AOC$.



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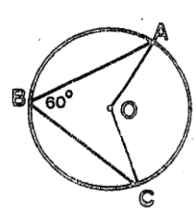
6. In the accompanying diagram of circle O, $m\angle ABC = 60°$. Find $m\angle AOC$.



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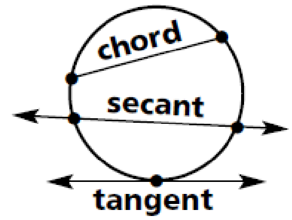
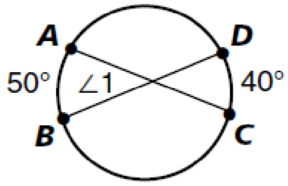
1. A **chord** is a segment whose endpoints are points on a circle.
A **secant** is a line that intersects a circle at two points.
A **tangent** is a line that intersects a circle at exactly one point.



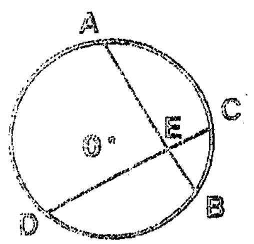


2. If two chords intersect *inside* a circle, the measure of each angle is equal to half the measure of the *sum* of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD}) = \frac{1}{2}(50^\circ + 40^\circ) = \frac{1}{2}(90^\circ) = 45^\circ$$

1 In the accompanying diagram, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\angle AED = 104^\circ$ and $m\overset{\frown}{CB} = 86^\circ$, find the $m\overset{\frown}{AD}$.

(1) 122° (3) 95°
(2) 52° (4) 86°



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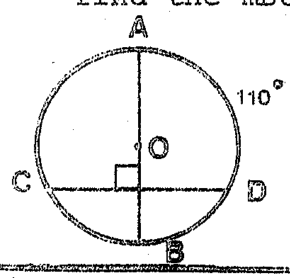
2. In circle O, diameter $\overline{AB} \perp \overline{CD}$. If $m\overset{\frown}{AD} = 110^\circ$, find the $m\overset{\frown}{BC}$.



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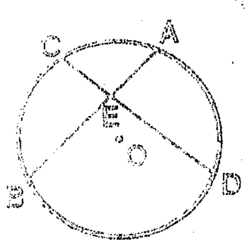
3. In circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\overset{\frown}{BC} = 50^\circ$ and $m\overset{\frown}{AD} = 90^\circ$, find the $m\angle AEC$.

(1) 70° (3) 135°
(2) 45° (4) 110°



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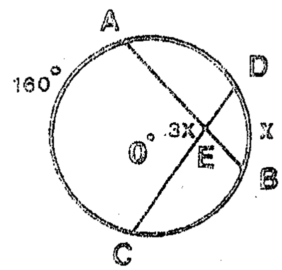
4. In the diagram below, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\angle AEC = 3x$, $m\overset{\frown}{AC} = 160^\circ$, and $m\overset{\frown}{DB} = x$, what is the value of x?

(1) 40° (3) 32°
(2) 96° (4) 80°



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5. Two chords intersecting within a circle form an angle whose measure is 70°. If one of the intercepted arcs measures 90°, what is the measure of the other intercepted arc?

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6. In a circle, chords $\overline{AB}$ and $\overline{CD}$ are perpendicular, and intersect at E. If $m\overset{\frown}{AC} = 70^\circ$, find the $m\overset{\frown}{BD}$.

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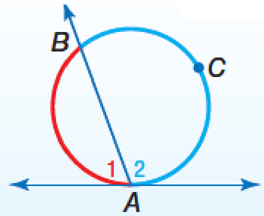
Use Intersecting Secants and Tangents

Words If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

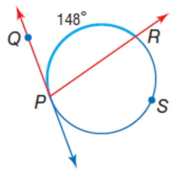
Example $m\angle 1 = \frac{1}{2}m\widehat{AB}$ and $m\angle 2 = \frac{1}{2}m\widehat{ACB}$



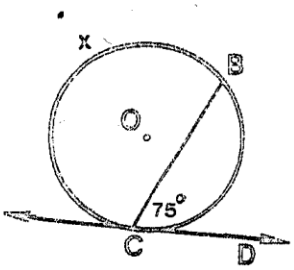
$m\angle QPR$

$m\angle QPR = \frac{1}{2}m\widehat{PR}$

$= \frac{1}{2}(148)$ or 74



1. In circle O, $\overline{BC}$ is a chord and $\overleftrightarrow{CD}$ is a tangent. Find the value of x.



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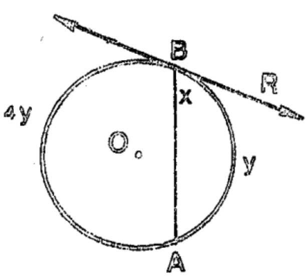
2. In circle O, $\overline{AB}$ is a chord and $\overleftrightarrow{BR}$ is a tangent. Find the value of x and y.



x = ________

y = ________

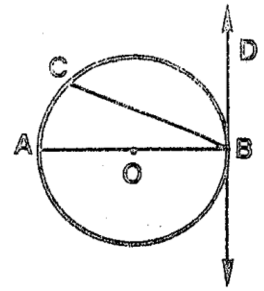
3. A regular pentagon RSTUV is inscribed in a circle. Find the measure of the acute angle formed by side $\overline{ST}$ and the tangent at T.

(1) 72° (3) 144°

(2) 36° (4) 18°

____

4. In the accompanying diagram, $\overleftrightarrow{BD}$ is tangent to circle O at B, $\overline{BC}$ is a chord, and $\overline{BOA}$ is a diameter. If $m\widehat{AC}:m\widehat{CB} = 1:5$, find the m∠DBC.



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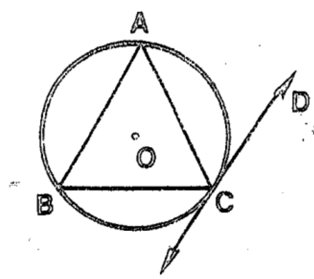
5. In circle O, AB = AC = BC and $\overleftrightarrow{CD}$ is a tangent at C. Find m∠BCD.



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6. Triangle DEF is inscribed in circle O. $m\widehat{DE}:m\widehat{EF}:m\widehat{DF} = 2:3:4$. Find the measure of the acute angle formed by side $\overline{EF}$ and the tangent to the circle at F.

(1) 40° (3) 80°

(2) 60° (4) 100°

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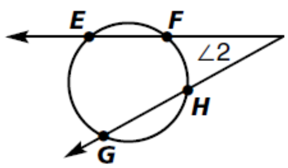
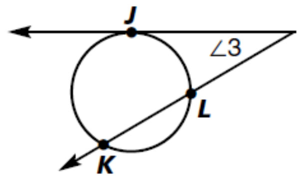
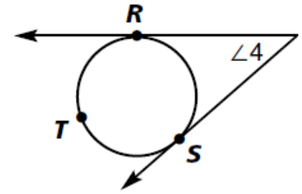
If two secants, a secant and a tangent, or two tangents intersect *outside* a circle, the measure of the angle formed is equal to half the measure of the *difference* of the intercepted arcs.





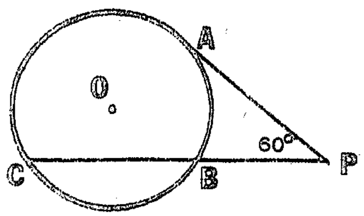


$m\angle 2 = \frac{1}{2}(m\overset{\frown}{EG} - m\overset{\frown}{FH})$ $m\angle 3 = \frac{1}{2}(m\overset{\frown}{JK} - m\overset{\frown}{JL})$ $m\angle 4 = \frac{1}{2}(m\overset{\frown}{RTS} - m\overset{\frown}{RS})$

1 Two tangents to a circle from an external point intercept a major arc of 280°. Find the number of degrees in the angle formed by the two tangents.

____

2. In the accompanying diagram, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn to circle O. If $m\overset{\frown}{AC}$ is four times $m\overset{\frown}{AB}$ and $m\angle P = 60°$, find $m\overset{\frown}{AB}$.



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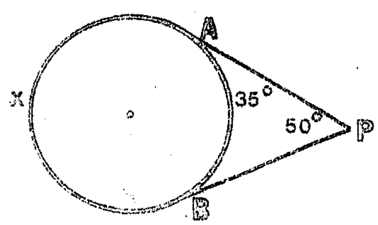
3. In the diagram below, $\overline{PA}$ and PB are tangents to circle O. Find the value of x.



(1) 135° (3) 50°

(2) 85° (4) 15°

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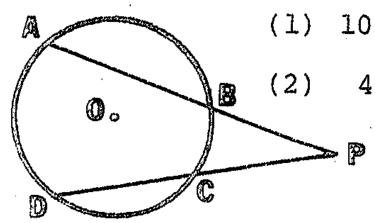
4. In the accompanying diagram, $\overline{PBA}$ and $\overline{PCD}$ are secants to the circle. If $m\overset{\frown}{AD} = 140°$, and $m\overset{\frown}{BC} = 60°$, find $m\angle P$.

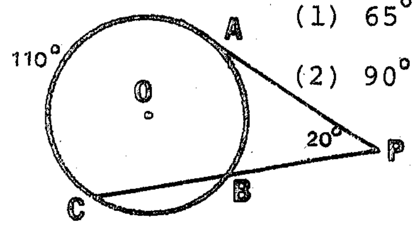


(1) 100° (3) 70°

(2) 40° (4) 30°

____

5. In the accompanying diagram, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn to circle O from point P. If $m\overset{\frown}{AC} = 110°$ and $m\angle P = 20°$, find the $m\overset{\frown}{AB}$.



(1) 65° (3) 70°

(2) 90° (4) 40°

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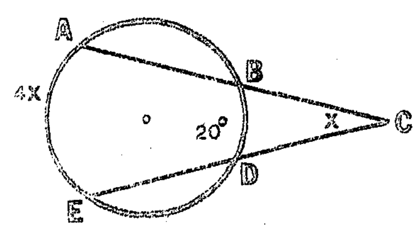
6. In the diagram below, secants $\overline{CBA}$ and $\overline{CDE}$ are drawn to circle O from point C. Find the value of x.



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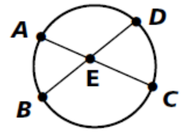
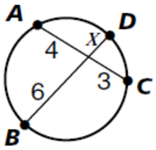
If two chords intersect, the product of the segment lengths along one chord is equal to the product of the segment lengths along the other chord.

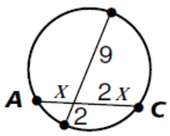


$AE \cdot EC = BE \cdot ED$

Examples: Find the value of x and the length of $\overline{AC}$.



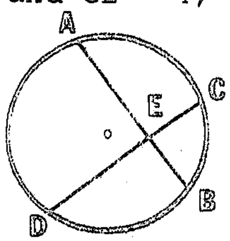
$4 \cdot 3 = 6 \cdot x$
$12 = 6x$
$2 = x$
$AC = 4 + 3 = 7$



$x \cdot 2x = 2 \cdot 9$
$2x^2 = 18$
$x^2 = 9$
$x = 3$

Check: $3 \cdot 2(3) \stackrel{?}{=} 2 \cdot 9$
$3 \cdot 6 \stackrel{?}{=} 2 \cdot 9$
$18 = 18$
$AC = x + 2x = 3 + 6 = 9$

1. In the accompanying figure, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If CD = 13, EB = 3, and CE = 4, find AE.



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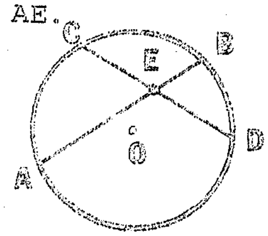
4. In the accompanying diagram, chords $\overline{AB}$ and $\overline{CD}$ of circle O intersect at E. If AE = x, EB = x-8, CE = 5, and ED = 4, find AE.



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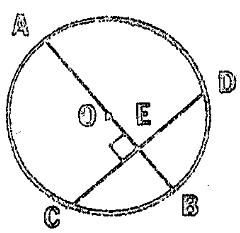
2. In circle O, diameter $\overline{AB}$ is perpendicular to chord $\overline{CD}$ at E. If AE = 24 and EB = 6, what is CD?

(1) 12 (3) 18

(2) 15 (4) 24



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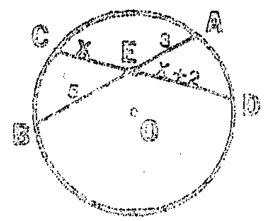
5. In the accompanying diagram, $\overline{AB}$ and $\overline{CD}$ are chords of the circle and intersect at E. If AE = 3, EB = 5, CE = x, and ED = x+2, find the value of x.



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3. In circle O, chords $\overline{AB}$ and $\overline{CD}$ interset at P. If AP = x, PB = y, and CP = z, what is the length of PD in terms of x, y, and z?

(1) $\frac{xy}{z}$ (3) $\frac{yz}{x}$

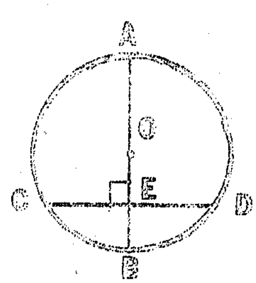
(2) $\frac{xz}{y}$ (4) $\frac{x+y}{z}$

____

6. In the accompanying figure, diameter $\overline{AB}$ is perpendicular to chord $\overline{CD}$ at E. If CE = 8, and EB = 4, find AE.

(1) 10 (3) 16

(2) 2 (4) 4



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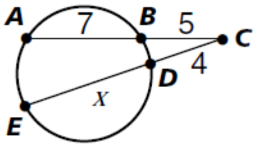
If two secant segments share the same point outside a circle, the product of the length of one secant and its external segment length is equal to the product of the length of the other secant and its external segment length.



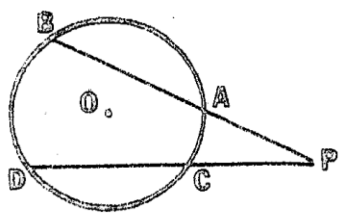
$AC \cdot BC = EC \cdot DC$

Example:
Given: $AB = 7$, $BC = 5$, $DC = 4$. Find the length of $\overline{ED}$ and $\overline{EC}$.

$$AC \cdot BC = EC \cdot DC$$
$$(7 + 5) \cdot 5 = (x + 4) \cdot 4$$
$$60 = 4x + 16$$
$$44 = 4x$$
$$11 = x$$

$ED = 11$; $EC = 15$

1. In the accompanying figure, secants $\overline{PAB}$ and $\overline{PCD}$ are drawn to circle O from P. If PA = 3, AB = 9, and PD = 18, find PC



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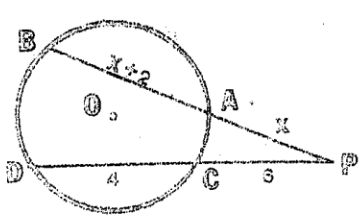
2. Two secants, $\overline{ABC}$ and $\overline{ADE}$ are drawn to a circle from external point A. If AB = 5, BC = 7, and AD = 4, find DE.

(1) 15 (3) 3

(2) 14 (4) 11

____

3. From point P outside of circle O, secants $\overline{PAB}$ and $\overline{PCD}$ are drawn. If PA = x, AB = x+2, PC = 6, and CD = 4, find the value of x.



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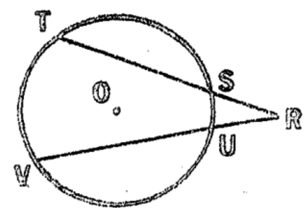
4. In the accompanying diagram, secants $\overline{RST}$ and $\overline{RUV}$ are drawn to circle O from R. If RT = 18, RS = 3, and RV = 27, find VU.

(1) 25 (3) 3

(2) 2 (4) 29



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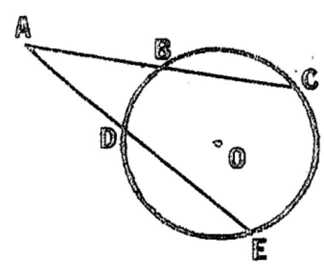
5. Two secants, $\overline{ABC}$ and $\overline{ADE}$ are drawn to a circle from external point A. If AB = 7, BC = 1, and AD = 4, find DE.



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6. Two secants, $\overline{PDQ}$ and $\overline{PBA}$ are drawn to a circle from external point P. If PQ = x, PD = y, and PA = z, find PB in terms of x, y, and z.

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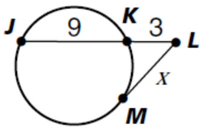
2. If a secant segment and a tangent segment share the same external point, the segment lengths follow a similar product rule.

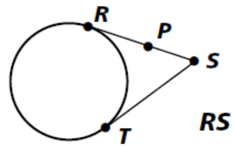


JL* · *KL* = *ML* · *ML

Example:
Given: $JK = 9$, $KL = 3$.
Find the length of $\overline{ML}$.

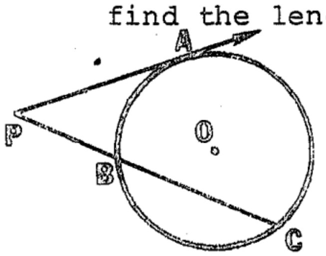
$$JL \cdot KL = ML \cdot ML$$
$$(9 + 3) \cdot 3 = x \cdot x$$
$$36 = x^2$$
$$6 = x \qquad ML = 6$$

3. If two tangent segments share the same external point, they are congruent.



RS* = *TS

1. In the accompanying figure, $\overrightarrow{PA}$ is tangent to circle O at A and $\overline{PBC}$ is a secant. If PC = 25, and PB = 4, find the length of PA.



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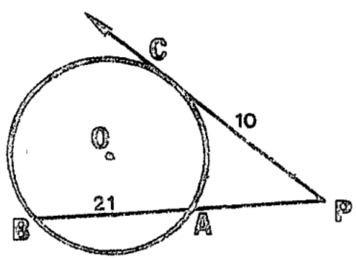
2. In the accompanying diagram, $\overrightarrow{PC}$ is tangent to circle O at C and $\overline{PAB}$ is a secant. If PC = 10, and AB = 21, find PA.



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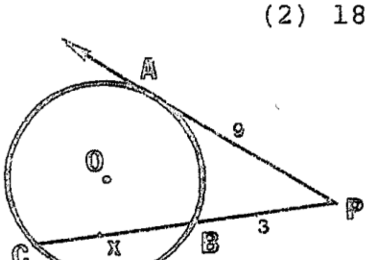
3. In the accompanying diagram, $\overrightarrow{PA}$ is tangent to circle O at A and $\overline{PBC}$ is a secant. If PA = 9, and PB = 3, find BC.

(1) 27 (3) 3

(2) 18 (4) 24



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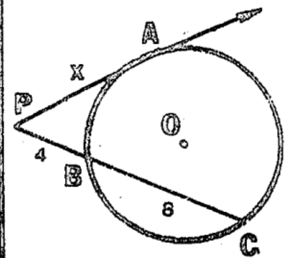
4. In the accompanying figure, $\overrightarrow{PA}$ is tangent to circle O at A and $\overline{PBC}$ is a secant. If PB = 4, and BC = 8 find PA.

(1) $2\sqrt{3}$ (3) $4\sqrt{2}$

(2) $4\sqrt{3}$ (4) $2\sqrt{2}$



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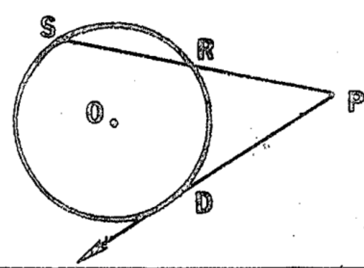
5. In the accompanying diagram, $\overrightarrow{PD}$ is tangent to circle O at D and $\overline{PRS}$ is a secant. If PD = x, PR = 12, and RS = x−3 find the value of x.



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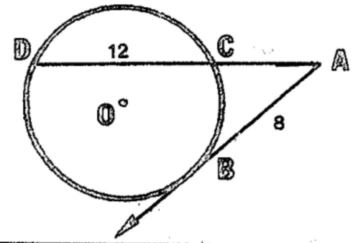
6. In the accompanying figure, $\overrightarrow{AB}$ is tangent to circle O at B and $\overline{ACD}$ is a secant. If DC = 12, and AB = 8, find AC.



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