11.7 Equations of Circles

Goal

Write and graph the equation of a circle.

Key Words

- standard equation of a circle

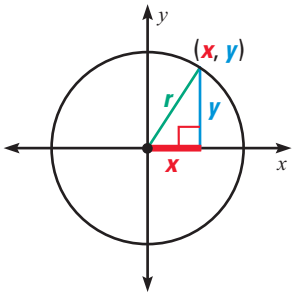
In the circle below, let point (x, y) represent any point on the circle whose center is at the origin. Let r represent the radius of the circle.

In the right triangle,

r = length of hypotenuse,

x = length of a leg,

y = length of a leg.



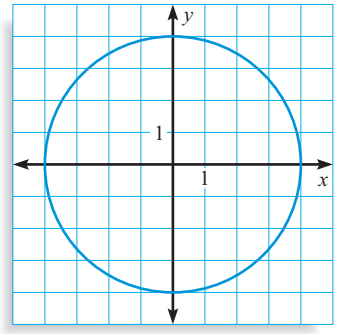
By the Pythagorean Theorem, you can write

$$x^2 + y^2 = r^2.$$

This is an equation of a circle with center at the origin.

EXAMPLE 1 Write an Equation of a Circle

Write an equation of the circle.



Solution

The radius is 4 and the center is at the origin.

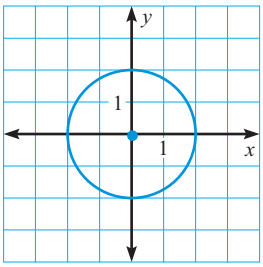
$x^2 + y^2 = r^2$	Write an equation of a circle with center at the origin.
$x^2 + y^2 = 4^2$	Substitute 4 for r.
$x^2 + y^2 = 16$	Simplify.

ANSWER An equation of the circle is $x^2 + y^2 = 16$.

Checkpoint Write an Equation of a Circle

Write an equation of the circle.

1.



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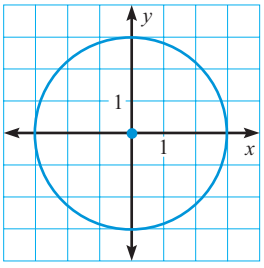
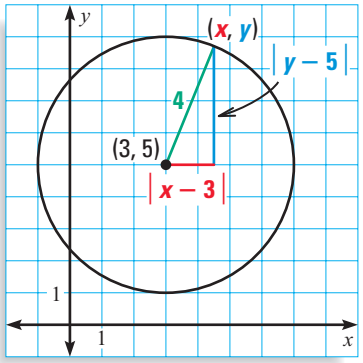


Student Help

SKILLS REVIEW
To review absolute value, see p. 662.

Standard Equation of a Circle If the center of a circle is not at the origin, you can use the Distance Formula to write an equation of the circle.

For example, the circle shown at the right has center (3, 5) and radius 4.



Let (x, y) represent any point on the circle. Use the Distance Formula to find the lengths of the legs.

leg: $|x - 3|$

leg: $|y - 5|$

hypotenuse: 4

Use these expressions in the Pythagorean Theorem to find an equation of the circle.

$$(x - 3)^2 + (y - 5)^2 = 4^2$$

This is an example of the **standard equation of a circle**.

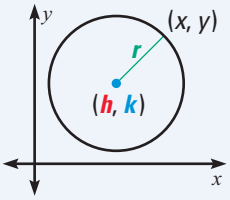
STANDARD EQUATION OF A CIRCLE

In the coordinate plane, the standard equation of a circle with center at (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

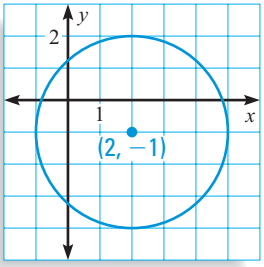
h: *x*-coordinate of the center

k: *y*-coordinate of the center



EXAMPLE 2 Write the Standard Equation of a Circle

Write the standard equation of the circle with center $(2, -1)$ and radius 3.



Solution

$(x - h)^2 + (y - k)^2 = r^2$ — Write the standard equation of a circle.

$(x - 2)^2 + (y - (-1))^2 = 3^2$ — Substitute 2 for h, -1 for k, and 3 for r.

$(x - 2)^2 + (y + 1)^2 = 9$ — Simplify.

ANSWER The standard equation of the circle is $(x - 2)^2 + (y + 1)^2 = 9$.

EXAMPLE 3 Graph a Circle

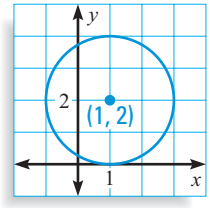
Graph the given equation of the circle.

a. $(x - 1)^2 + (y - 2)^2 = 4$

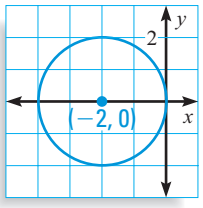
b. $(x + 2)^2 + y^2 = 4$

Solution

a. Rewrite the equation of the circle as $(x - 1)^2 + (y - 2)^2 = 2^2$. The center is $(1, 2)$ and the radius is 2.



b. Rewrite the equation of the circle as $(x - (-2))^2 + (y - 0)^2 = 2^2$. The center is $(-2, 0)$ and the radius is 2.



Checkpoint Circles Not Centered at the Origin

3. Write the standard equation of the circle with center (–4, –6) and radius 5.

Graph the given equation of the circle.

4. $(x - 1)^2 + y^2 = 25$

5. $(x + 2)^2 + (y - 4)^2 = 16$

11.7 Exercises

Guided Practice

Vocabulary Check

1. Which of the following is a *standard equation of a circle*?

A. $(x+ 2)^2 = 16y$

B. $(x^2 - 5) + (y^2 - 8) = 16$

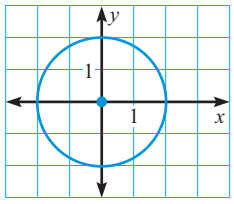
C. $(x - 4)^2 + (y - 3)^2 = 16$

D. $2x^2 + 3y - 5 = 16$

Skill Check

Give the radius and the coordinates of the center. Write the equation of the circle in standard form.

2.



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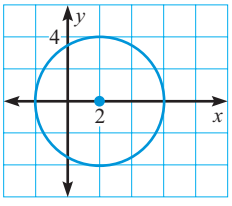

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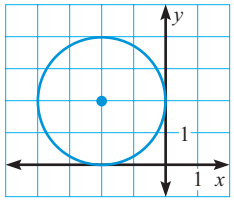


Practice and Applications

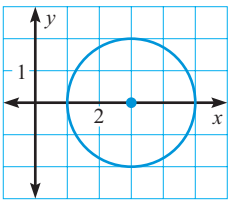
Extra Practice

See p. 696.

Matching Equations Match each graph with its equation.

A. $x^2 + y^2 = 4$ **B.** $(x - 3)^2 + y^2 = 4$ **C.** $(x + 3)^2 + y^2 = 4$

5.



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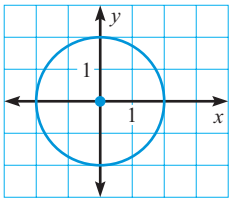

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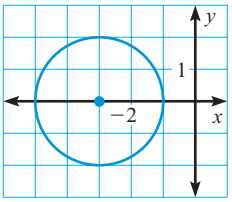


Student Help
CLASSZONE.COM

Homework Help
Extra help with problem solving in Exs. 8–15 is at classzone.com

Using Standard Equations Give the radius and the coordinates of the center of the circle with the given equation. Then graph the circle.

8. $x^2 + y^2 = 36$

9. $x^2 + y^2 = 1$

10. $(x - 2)^2 + (y - 6)^2 = 49$

11. $(x - 4)^2 + (y - 3)^2 = 16$

12. $(x - 5)^2 + (y - 1)^2 = 25$

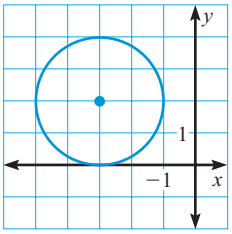
13. $(x + 2)^2 + (y - 3)^2 = 36$

14. $(x - 2)^2 + (y + 5)^2 = 4$

15. $x^2 + (y - 5)^2 = 64$

Using Graphs Give the radius and the coordinates of the center of the circle. Then write the standard equation of the circle.

16.



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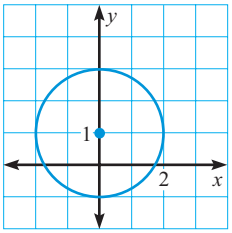

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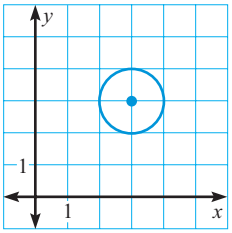

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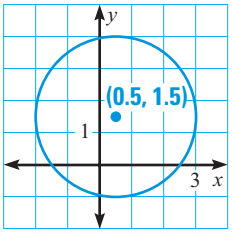

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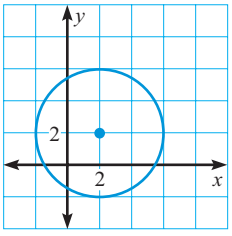

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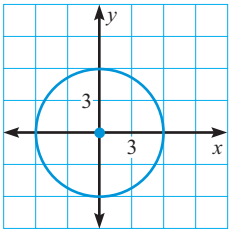


Homework Help

Example 1: Exs. 5–7, 21
Example 2: Exs. 5–7, 16–27
Example 3: Exs. 8–15

Writing Equations Write the standard equation of the circle with the given center and radius.

22. center (0, 0), radius 10

23. center (4, 0), radius 4

24. center (3, −2), radius 2

25. center (−1, −3), radius 6

26. center (−3, 5), radius 3

27. center (1, 0), radius 7

EXAMPLE Use the Equation of a Circle

The equation of a circle is $(x - 5)^2 + (y - 1)^2 = 9$. Without sketching the circle, tell whether the point is *on* the circle, *inside* the circle, or *outside* the circle.

a. (6, 0) **b.** (8, 2)

Solution

Substitute the coordinates of the point into the equation.

If the left side is *less than* the right side, the point is *inside* the circle.

If the left side is *greater than* the right side, the point is *outside* the circle.

a. $(x - 5)^2 + (y - 1)^2 = 9$

$(6 - 5)^2 + (0 - 1)^2 \stackrel{?}{=} 9$

$1^2 + (-1)^2 \stackrel{?}{=} 9$

$2 < 9$

Because $2 < 9$, the point (6, 0) is *inside* the circle.

b. $(x - 5)^2 + (y - 1)^2 = 9$

$(8 - 5)^2 + (2 - 1)^2 \stackrel{?}{=} 9$

$3^2 + 1^2 \stackrel{?}{=} 9$

$10 > 9$

Because $10 > 9$, the point (8, 2) is *outside* the circle.

Student Help

STUDY TIP

If the left side is *equal* to the right side, the point is *on* the circle.

Equation of a Circle **The equation of a circle is $(x - 2)^2 + (y + 3)^2 = 4$. Tell whether the point is *on* the circle, *inside* the circle, or *outside* the circle. Use the example above as a model.**

28. $R(0, 0)$ **29.** $A(2, -4)$ **30.** $X(0, -3)$ **31.** $K(3, -1)$

32. $M(1, -4)$ **33.** $T(2, -5)$ **34.** $D(2, 0)$ **35.** $Z(2.5, -3)$

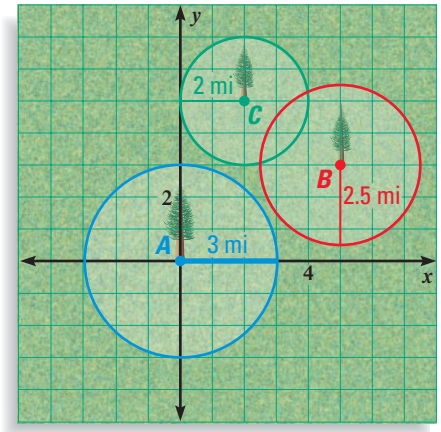
Link to Communications

CELL PHONE towers are sometimes built to look like trees so that they blend in with their environment. Other cell phone towers have also been built to resemble farm silos and cactus plants.

Cell Phones **In Exercises 36 and 37, use the following information.** A cellular phone network uses towers to transmit calls. Each tower transmits to a circular area. On a grid of a town, the coordinates of the towers and the circular areas covered by the towers are shown.

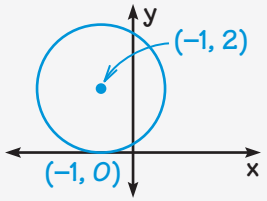
36. Write the equations that represent the transmission boundaries of the towers.

37. Tell which towers, if any, transmit to phones located at $J(1, 1)$, $K(4, 2)$, $L(3.5, 4.5)$, $M(2, 2.8)$, and $N(1, 6)$.



38. Error Analysis A student was asked to write the standard equation of the circle below. Why is the equation incorrect?

center $(-1, 2)$
radius 2
$(x - 1)^2 + (y + 2)^2 = 2$ ✗



Challenge **Use the given information to write the standard equation of the circle.**

39. The center is $(1, 2)$. A point on the circle is $(4, 6)$.

40. The center is $(3, 2)$. A point on the circle is $(5, 2)$.

Standardized Test Practice

41. Multiple Choice What is the standard form of the equation of a circle with center $(-3, 1)$ and radius 2?

Ⓐ $(x - 3)^2 + (y - 1)^2 = 2$ Ⓑ $(x + 3)^2 + (y - 1)^2 = 2$

Ⓒ $(x - 3)^2 + (y - 1)^2 = 4$ Ⓓ $(x + 3)^2 + (y - 1)^2 = 4$

42. Multiple Choice The center of a circle is $(-3, 0)$ and its radius is 5. Which point does *not* lie on the circle?

Ⓕ $(2, 0)$ Ⓖ $(0, 4)$ Ⓗ $(-3, 0)$ Ⓙ $(-3, -5)$

Mixed Review

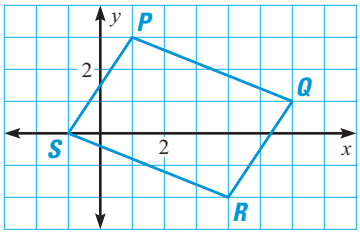
Finding an Image **Find the coordinates of *P′*, *Q′*, *R′*, and *S′*, using the given translation.** *(Lesson 3.7)*

43. $(x, y) \rightarrow (x + 2, y)$

44. $(x, y) \rightarrow (x - 4, y + 1)$

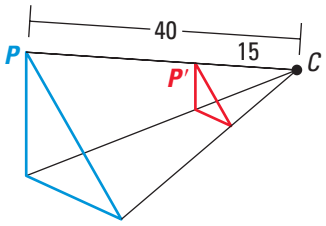
45. $(x, y) \rightarrow (x - 1, y - 1)$

46. $(x, y) \rightarrow (x + 3, y + 6)$

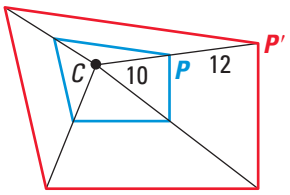


Identifying Dilations **Tell whether the dilation is a *reduction* or an *enlargement*. Then find its scale factor.** *(Lesson 7.6)*

47.



48.



Algebra Skills

Solving Equations **Solve the equation.** *(Skills Review, p. 673)*

49. $14 = -3x - 7$ **50.** $11 - x = -2$ **51.** $20 = 5x - 12 - x$