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|  | Exponential Growth and Decay  |

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|  Exponential functions are of the form      | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec29.gif |

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| **Notice:** The **variable *x* is an exponent**.  As such, the graphs of these functions are not straight lines.  In a straight line, the "rate of change" is the same across the graph.  In these graphs, the "rate of change" increases or decreases across the graphs. | http://www.regentsprep.org/regents/math/algebra/ae7/boatlooker.gif |

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  Observe how the graphs of exponential functions change based upon the values of *a* and *b:*

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| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec29.gif |

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| Example: | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec30.gif |

when *a* > 0 and the *b* is between 0 and 1, the graph will be decreasing (decaying).For this example, each time *x* is increased by 1, y decreases to one half of its previous value.http://www.regentsprep.org/regents/math/algebra/ae7/fixpic1.gifSuch a situation is called **Exponential Decay.**  |

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| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec29.gif |

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| Example: | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec31.gif |

when *a* > 0 and the *b* is greater than 1, the graph will be increasing (growing).For this example, each time *x* is increased by 1, y increases by a factor of 2.http://www.regentsprep.org/regents/math/algebra/ae7/fixpic2.gifSuch a situation is called **Exponential Growth.** |

 Many real world phenomena can be modeled by functions that describe how things grow or decay as time passes.  Examples of such phenomena include the studies of populations, bacteria, the AIDS virus, radioactive substances, electricity, temperatures and credit payments, to mention a few.Any quantity that grows or decays by a fixed percent at regular intervals is said to possess **exponential growth**or**exponential decay.**At the Algebra level, there are two functions that can be easily used to illustrate the concepts of growth or decay in applied situations.  When a quantity grows by a fixed percent at regular intervals, the pattern can be represented by the functions,

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| **Growth:**http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec13.gif |

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| **Decay:**http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec20.gif |

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|   | *a* = initial **amount** before measuring growth/decay*r* = growth/decay **rate** (often a percent)*x* = number of **time** intervals that have passed |

**Example:** A bank account balance, *b,* for an account starting with *s* dollars, earning an annual interest rate, *r,*and left untouched for *n* years can be calculated as  http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec46.gif *(an exponential growth formula).*   Find a bank account balance to the *nearest dollar*, if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

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| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec47.gif | http://www.regentsprep.org/regents/math/algebra/ae7/money.gif |

 We will now examine rate of growth and decay in a three step process.  We will (1) build a chart to examine the data and "see" the growth or decay, (2) write an equation for the function, and (3) prepare a scatter plot of the data along with the graph of the function.Consider these examples of growth and decay:

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| **Growth:http://www.regentsprep.org/regents/math/algebra/ae7/cellphone.gif  Cell Phone Users** | In 1985, there were 285 cell phone subscribers in the small town of Centerville.  The number of subscribers **increased**by 75% per year after 1985.  How many cell phone subscribers were in Centerville in 1994?(Don't consider a fractional part of a person.) |

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| Years | *x* = 11986 | 21987 | 31988 | 41989 | 51990 | 61991 | 71992 | 81993 | 91994 |
| Number of Cell Phone users | 498 | 872 | 1527 | 2672 | 4677 | 8186 | 14325 | 25069 | **43871** |

**There are 43871 subscribers in 1994.**

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| Function:http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec13.gif | *a* = the initial amount before the growth begins*r* = growth rate*x* = the number of intervals |
| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec21.gif | as *x* ranges from 1 to 9 for this problem |

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| The scatter plot of the data table can be prepared by hand or with the use of a graphing calculator.  For graphing the function, employ your graphing calculator.

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| http://www.regentsprep.org/regents/math/algebra/ae7/84seblank.gif | See how to prepare a scatter plot of your data table using your TI 83+/84+ graphing calculator.  [Click here.](http://mathbits.com/MathBits/TISection/statistics1/scatterplot.htm)After the data points are plotted, set Y1 = to the function, and graph.  The function and the scatter plot will overlap as they did at the right. |

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 | http://www.regentsprep.org/regents/math/algebra/ae7/cellphonegraph.gifhorizontal axis = year (1986 = 1)vertical axis = number of cell phone users |

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| **Growth by doubling:****Bacteria**

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| http://www.regentsprep.org/regents/math/algebra/ae7/bug.gif |

 | One of the most common examples of exponential growth deals with bacteria.  Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling.  For example, if we start with only one bacteria which can double every hour, by the end of one day we will have over 16 million bacteria. |

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| End of  Hour | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | ... | 24 |
| Bacteria - starting with one | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | 4096 | 8192 | 16384 | ... | 16777216 |
| Pattern: | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 210 | 211 | 212 | 213 | 214 |   | 224 |

 |
| **At the end of 24 hours, there are 16,777,216 bacteria.**By looking at the pattern, we see that the growth in this situation can be represented as a function: http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec1.gif  Will our formula show this same function?  If an amount doubles, the rate of increase is 100%.

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| Function:http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec13.gif | *a* = the initial amount before the growth begins*r* = growth rate*x* = the number of intervals |
| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec22.gif | as *x* ranges from 1 to 24 for this problem |

*horizontal axis = end of hourvertical axis = number of bacteria* |
| http://www.regentsprep.org/regents/math/algebra/ae7/graphbacteria1.gif |   | http://www.regentsprep.org/regents/math/algebra/ae7/graphbacteria2.gif |
| Let's examine the graph of our scatter plot and function.  To the left of the origin we see that the function graph tends to flatten, but stays slightly above the *x*-axis.  To the right of the origin the function graph grows so quickly that it is soon off the graph.  The rate at which the graph changes increases as time increases. |   | When we can see larger *y*-values, we see that the growth still continues at a rapid rate.  This is what is meant by the expression "increases exponentially".  |
| **Note:** In reality, exponential growth does not continue indefinitely.  There would, eventually, come a time when there would no longer be any room for the bacteria, or nutrients to sustain them.  Exponential growth actually refers to only the early stages of the process and to the manner and speed of the growth. |

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| **Decay:  http://www.regentsprep.org/regents/math/algebra/ae7/tennispic.gif  Tennis Tournament** | Each year the local country club sponsors a tennis tournament.  Play starts with 128 participants.  During each round, half of the players are eliminated.  How many players remain after 5 rounds? |

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| Rounds | 1 | 2 | 3 | 4 | 5 |
| Number of Players left | 64 | 32 | 16 | 8 | 4 |

**There are 4 players remaining after 5 rounds.**

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| Function:http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec20.gif | *a* = the initial amount before the decay begins*r* = decay rate*x* = the number of intervals |
| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec23.gif | as *x* ranges from 1 to 5 for this problem |

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| Notice the shape of this graph compared to the graphs of the growth functions.  | http://www.regentsprep.org/regents/math/algebra/ae7/tennisgraph.gif*horizontal axis = roundsvertical axis = number of players left* |

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| **Decay by half-life:**http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec1.jpg | The pesticide DDT was widely used in the United States until its ban in 1972.  DDT is toxic to a wide range of animals and aquatic life, and is suspected to cause cancer in humans.  The *half-life*of DDT can be 15 or more years.  *Half-life*is the amount of time it takes for half of the amount of a substance to decay.  Scientists and environmentalists worry about such substances because these hazardous materials  continue to be dangerous for many years after their disposal. For this example, we will set the half-life of the pesticide DDT to be 15 years.**Let's mathematically examine the half-life of 100 grams of DDT.** |
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| End of Half life cycle |  **1**15 yrs |  **2**30 yrs | **3**45 yrs |  **4**60 yrs | **5**75 yrs | **6**90 yrs | **7**105 yrs |  **8**120 yrs | **9**135 yrs | **10**150 yrs |
| Grams of DDT remaining | 50 | 25 | 12.5 | 6.25 | 3.125 | 1.5625 | .78125 | .390625 | .1953125 | .09765625 |
| Pattern: | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec2.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec3.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec4.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec5.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec6.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec7.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec8.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec9.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec10.gif | http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec11.gif |

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|   By looking at the pattern, we see that this decay can be represented as a function:  http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec12.gif

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| Function:http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec20.gif | *a* = the initial amount before the decay begins*r* = decay rate*x* = the number of intervals |
| http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec24.gif | as *x* ranges from 1 to 10 for this problem |

http://www.regentsprep.org/regents/math/algebra/ae7/ExpDec25.gif*horizontal axis = end of half life cyclevertical axis = grams of DDT remaining* |

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| http://www.regentsprep.org/regents/math/algebra/ae7/graphnuclear1.gifLet's examine the scatter plot and the function.  At 0 the y-intercept is 100.  To the right of the origin we see that the graph declines rapidly and then tends to flatten, staying slightly above the *x*-axis.  The rate of change decreases as time increases. |   | http://www.regentsprep.org/regents/math/algebra/ae7/decaygraph2.gifWhen we zoom in on the flattened area of the graph, we see that the graph does stay above the x-axis.  This makes sense because we could not have a "negative" number of grams of DDT leftover. |

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 Exponential growth and decay are mathematical changes.  The rate of the change continues to either increase or decrease as time passes.  In exponential growth, the rate of change increases over time - the rate of the growth becomes faster as time passes.  In exponential decay, the rate of change decreases over time - the rate of the decay becomes slower as time passes.  Since the rate of change is not constant (the same) across the entire graph, these functions are not straight lines.

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| http://www.regentsprep.org/regents/math/algebra/ae7/DogsleepJ.gif |

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