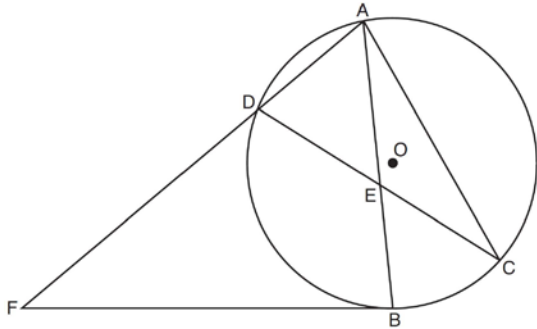## G.C.A.2: Chords, Secants and Tangents 18

1 Chords $\overline{AB}$ and $\overline{CD}$ intersect at E in circle O, as shown in the diagram below. Secant $\overline{FDA}$ and tangent $\overline{FB}$ are drawn to circle O from external point F and chord $\overline{AC}$ is drawn. The $m\overarc{DA} = 56$, $m\overarc{DB} = 112$, and the ratio of $m\overarc{AC} : m\overarc{CB} = 3 : 1$.

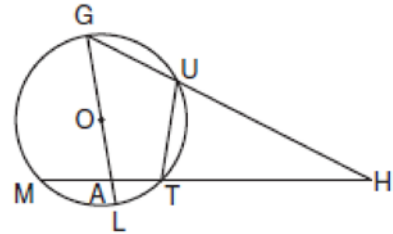Determine $m\angle CEB$. Determine $m\angle F$. Determine $m\angle DAC$.

2 In circle O shown below, diameter $\overline{AC}$ is perpendicular to $\overline{CD}$ at point C, and chords $\overline{AB}$, $\overline{BC}$, $\overline{AE}$, and $\overline{CE}$ are drawn.
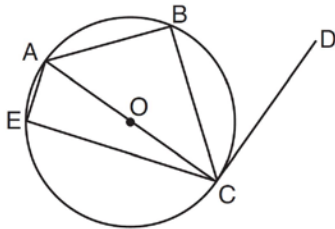

Which statement is *not* always true?
1) $\angle ACB \cong \angle BCD$
2) $\angle ABC \cong \angle ACD$
3) $\angle BAC \cong \angle DCB$
4) $\angle CBA \cong \angle AEC$

3 Given circle O with diameter $\overline{GOAL}$; secants $\overline{HUG}$ and $\overline{HTAM}$ intersect at point H; $m\overarc{GM} : m\overarc{ML} : m\overarc{LT} = 7 : 3 : 2$; and chord $\overline{GU} \cong$ chord $\overline{UT}$. Find the ratio of $m\angle UGL$ to $m\angle H$.

4 In the accompanying diagram, circle O has radius $\overline{OD}$, diameter $\overline{BOHF}$, secant $\overline{CBA}$, and chords $\overline{DHG}$ and $\overline{BD}$; $\overline{CE}$ is tangent to circle O at D; $m\overarc{DF} = 80$; and $m\overarc{BA} : m\overarc{AG} : m\overarc{GF} = 3 : 2 : 1$. Find $m\overarc{GF}$, $m\angle BHD$, $m\angle BDG$, $m\angle GDE$, $m\angle C$, and $m\angle BOD$.
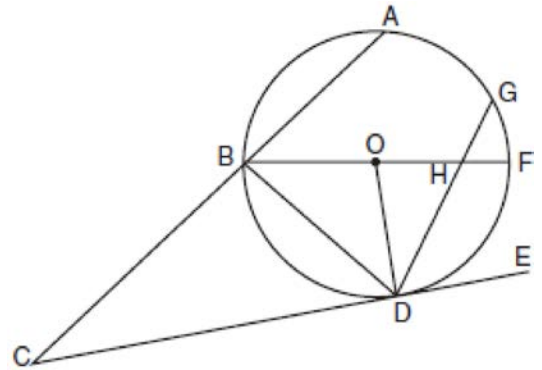
5   In the accompanying diagram, $\overline{PA}$ is tangent to circle O at A, chord $\overline{AC}$ and secant $\overline{PCED}$ are drawn, and chords $\overline{AOB}$ and $\overline{CD}$ intersect at E. If $m\overset{\frown}{AD} = 130$ and $m\angle BAC = 50$, find $m\angle P$, $m\angle BEC$, and $m\angle PCA$.
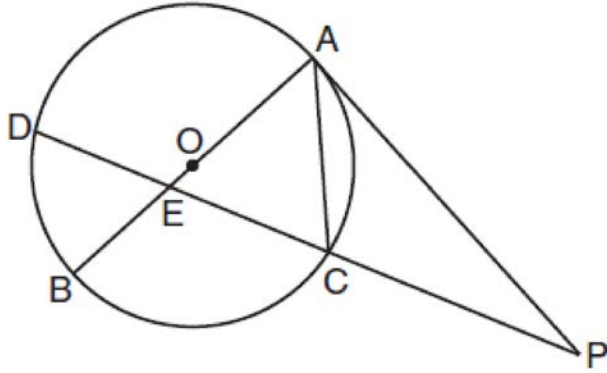
G.C.A.2: Chords, Secants and Tangents 18
Answer Section

1 ANS:

52, 40, 80. $360 - (56 + 112) = 192$. $\dfrac{192 - 112}{2} = 40$. $\dfrac{112 + 48}{2} = 80$

$$\dfrac{1}{4} \times 192 = 48$$

$$\dfrac{56 + 48}{2} = 52$$

REF: 081238ge

2 ANS: 1   REF: 061520geo

3 ANS:

2:1. $\overarc{GM}$ and $\overarc{ML}$ form a semi-circle and measure 126° $\left(\dfrac{7}{10} \times 180\right)$ and 54° $\left(\dfrac{3}{10} \times 180\right)$, respectively.

$\overarc{LT}$ measures 36°. $\overarc{GM}$ and $\overarc{ML}$ form a semi-circle and measure 126°. $\overarc{GUT}$ measures 144° $(180 - 36)$. Equal chords intercept equal arcs. Because chord $\overline{GU} \cong$ chord $\overline{UT}$, $\overarc{GU}$ and $\overarc{UT}$ each measures 72° $\left(\dfrac{144}{2}\right)$. $m\overarc{UTL} = 108$ $(72 + 36)$. The measure of an inscribed angle is half that of its intercepted arc. So $m\angle UGL = 54$. The angle formed by a tangent and a secant is equal to half the difference between the intercepted arcs. $\dfrac{126 - 72}{2} = 27$. The ratio of $m\angle UGL$ to $m\angle H$ is 54:27, or 2:1.
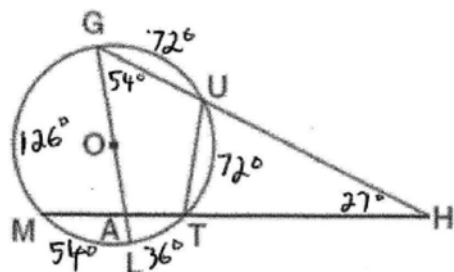
REF: 080333b

4   ANS:

30, 65, 75, 55, 35, 100. $\overset{\frown}{BA}$, $\overset{\frown}{AG}$ and $\overset{\frown}{GF}$ form a semi-circle and measure 90° ($\frac{3}{6} \times 180$), 60° ($\frac{2}{6} \times 180$) and

30° ($\frac{1}{6} \times 180$), respectively. The measure of an inscribed angle is half that of its intercepted arc. So

$m\angle BDG = 75$ ($\frac{90+60}{2}$) and $m\angle HBD = 40$ ($\frac{80}{2}$). Therefore $m\angle BHD = 65$ (180-(75+40)). The angle

formed by a tangent and a chord is half the intercepted arc. Since the intercepted arc is 110° (80+30),

$m\angle GDE = 55$. Given diameter $\overline{BOHF}$ and $m\overset{\frown}{DF} = 80$, $m\overset{\frown}{BD} = 100$. The angle formed by a tangent and a

secant is equal to half the difference between the intercepted arcs, so $m\angle C$. $\frac{(60+30+80)-100}{2} = 35$. The

measure of a central angle is equal to the measure of the arc it intercepts, so $m\angle BOD = 100$.
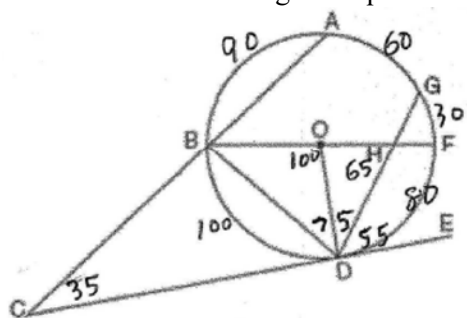
REF: 080633b

5   ANS:
25, 115, 115

REF: 011033b